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Comparative Physiology. — *Untersuchungen über Reaktionsverzögerung in Schneckenmuskeln und ihre Bedeutung für die Wechselwirkung zwischen Kontraktion und viskösem Tonus.* Von H. J. JORDAN. (Aus der zoologischen Station zu Neapel und dem Institut für vergleichende Physiologie der Universität Utrecht.)

(Communicated at the meeting of October 31, 1936).

Aufgabe der folgenden Untersuchungen ist, zu erforschen, was im Schneckenmuskel während der Dekreszente nach Reizung geschieht. Es gibt Forscher, die in einer flachen Dekreszente den Ausdruck von „Tonus“ sehen und glauben es gäbe bei den Schneckenmuskeln nur eine einzige Form von Tonus und dieser beruhe auf dem Festhalten einer Kontraktion, wie sie bei jeder Reizung von Muskeln auftritt. Auch die tonische Ruhedehnung sei eine flache Dekreszente. Wir werden zeigen, dass diese Auffassung falsch ist und dass eine flache Dekreszente verschiedene Ursachen haben kann. Das Auffinden dieser Ursachen gewährt einen neuen Einblick in das Wesen des Zusammenarbeitens von Tonus und Bewegung.

I. „Reiztonuseffekt“ bei *Aplysia limacina* und *A. depilans* (s. JORDAN 1933).

Es ist möglich am Fusse von *Aplysia limacina* und *A. depilans* Kontraktionen mit flacher Dekreszente hervorzurufen, die der plastisch-tonischen Ruhedehnung ähnlich sind. Dies gelingt am leichtesten, wenn der Aplysienfuss während der Kontraktion schwer belastet ist. Diese geringe Neigung der Dekreszenten nach Reizung wird aber durch ganz andere Ursachen hervorgerufen als beim Tonus.

In der flachen Dekreszente drückt sich lediglich eine stark verzögerte, die Reizung überdauernde, *spontane* Erschlaffung aus. Das Gewicht am Schreibhebel dient lediglich dazu, durch Auseinanderziehen des Muskels, die Erschlaffung sichtbar zu machen. Wir beweisen unsere Behauptung dadurch, dass wir bei Kontraktionen mit flacher, scheinbar tonischer Dekreszente auf dem Gipfel der Kontraktion eine Pause von beispielsweise 5 Minuten machen und zwar entweder durch Bremsung, d.h. Fixierung beider Muskelenden, oder durch zeitweilige Entlastung. Nach Entbremsung oder Neubelastung fällt bei *Aplysia* die Kurve stets steil ab, wie bei einer schnellen Kontraktion. (Fig. 1.)¹⁾ Eine schnelle Erschlaffung weicht nur

¹⁾ Dies gilt auch für den Retractor pharyngis von *Helix*, der kein Hohlmuskel ist. Zu beachten ist, dass der Begriff „schnell“ andere Grössenordnung hat als in der Physiologie quergestreifter Muskeln. Das Kymographion dreht sich mit einer Geschwindigkeit von 1 cm in der Minute.

wenige Winkelgrade von der Ordinate ab. Folgende Zahlenbeispiele mögen das Gesagte illustrieren.

Aplysia depilans, Fuss mit 17 gr belastet; Temperatur 4° C.

- a. 1. Der letzte Teil der, dem Ruhetonus entsprechenden Dehnungskurve, vor den Reizversuchen verläuft unter einem Winkel mit der Abszisse von 62.3°
 2. Dekreszente einer Kontraktionskurve (starke faradische Reizung) verläuft unter einem Winkel mit der Abszisse von 72.3°
 3. Dekreszente nach gleicher Reizung, aber nach Pause (Bremse) auf dem Gipfel von 5 Minuten. Winkel mit der Abszisse von 87.3°
 - b. 1. Dekreszente ohne Gipfelpause. Winkel mit der Abszisse von 78°.
 2. Dekreszente mit Gipfelpause von 5 Minuten (durch Entlastung). Winkel mit der Abszisse von 88°.
- (Fig. 1.)

Während der Pause würde visköser Tonus sich unverändert behaupten, während verzögerte Erschlaffung ununterbrochen, wenn auch an sich unsichtbar, weiter geht. Der Tonus quergestreifter Muskeln muss dauernd durch rhythmische Erregung unterhalten werden; wir nennen ihn darum „Tetanotonus“. Hier, bei *Aplysia*, überdauert ein tonusähnlicher Zustand die Reizung; wir sprechen von „Reiztonuseffekt“: beide Erscheinungen stehen gegenüber dem plastischen oder viskösen Tonus, der sich unabhängig von Erregung behauptet und Stunden- ja selbst Tagelang keine Erschlaffung erkennen lässt. Es ist mir bei *Aplysia* niemals gelungen einen, dem viskösen Tonus ähnlichen Zustand der Muskulatur durch elektrische Reizung zu erzielen¹⁾.

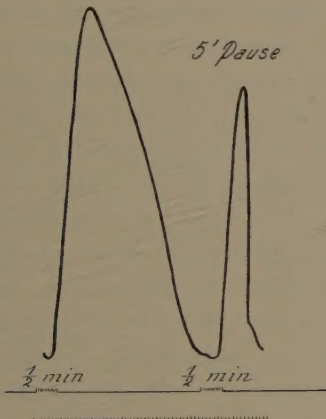


Fig. 1. *Aplysia limacina*. Reiztonuseffekt und seine Aufhebung durch Gipfelpause. Zwei Kontraktionskurven vom Fusse, je nach sehr starker faradischer Reizung von $\frac{1}{2}$ Minute Dauer. Last 17 gr, Temp. 4°. Erste Kurve mit stark „tonischer“ Dekreszente (Reiztonuseffekt). Zweite Kurve: auf dem Gipfel wird der Fuss 5 Minuten lang entlastet, dann wieder voll mit 17 gr belastet: steile Dekreszente. Zeit in $\frac{1}{10}$ Minuten.

Dass der Reiztonuseffekt nichts mit dem plastischen Tonus zu schaffen hat, kann man durch Versuche mit Magnesiumsulfat beweisen. Dieses Salz vernichtet, wie ich gezeigt habe (JORDAN, 1930), die langsame Kontraktion und den viskösen Tonus, während die schnelle Kontraktion und der Reiztonuseffekt unvermindert bestehen bleiben. Tyramin ist bei den Cephalopoden nach SERENI das tonuserzeugende Hormon. Bei diesen Tieren kommt visköser Tonus nicht vor, nur Tetanotonus. Entsprechend

hat Tyramin denn auch keinerlei Einfluss auf den viskösen Tonus von

¹⁾ Ueber tonische Kontraktionen bei *Aplysia* und die spezifische Methodik um sie auszulösen habe ich früher berichtet (JORDAN 1930, 1935a).

Aplysia; es erzeugt (oder verstärkt) hier dagegen starken Reiztonus nach faradischer Reizung.

Die Frage ob der Reiztonuseffekt auf normalen Muskeleigenschaften beruht oder ob er Ausdruck einer Schädigung durch starke Reizung ist, lässt sich nicht entscheiden. Eine biologische Auswertung dieser Erscheinung liegt immerhin im Bereiche der Möglichkeit.

II. Der Fuss von *Helix pomatia*. Wechselwirkung zwischen Kontraktion und viskösem Tonus.

Bei *Helix* kann sich in der Dekreszente nach Reizung wesentlich komplizierteres Verhalten ausdrücken als bei *Aplysia*. Sie kann als reine Erschlaffungskurve verlaufen oder aber mehr oder weniger durch viskösen Tonus kompliziert sein.

Wir untersuchen dieses durch Anwendung von Belastungspausen, sowie durch die folgende Technik. Widerstand, verursacht durch viskösen Tonus,

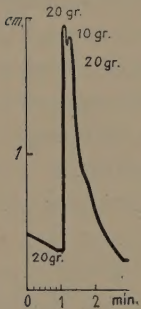


Fig. 2. *Helix pomatia*, Fuss. Reine schnelle Kontraktion. Entlastung von 20 auf 10 gr hat keinen Schneepflugeffekt zur Folge. Ordinate: effektive Längenänderungen des Fusses. Reiz R.A. 12 cm, 1 Sek. lang.

gibt sich bei seiner Ueberwindung durch „Schneepflugeffekt“ zu erkennen. Wenn nämlich die Dehnung unter Verschiebung von Intermizellarsubstanz stattgefunden hat, folgt bei partieller Entlastung (in unseren Fällen von 20 auf 10 gr) eine kürzere oder längere horizontale Kurvenstrecke, die spontan, ohne Gewichtserhöhung, wieder in Dehnung übergeht. In früheren Publikationen (JORDAN 1934, 1935b und 1936) habe ich gezeigt, dass „Schneepflugeffekt“ sowohl bei den uns beschäftigenden Muskeln, als bei nicht vulkanisiertem, plastiziertem Kautschuk bei passiver Dehnung entstehen kann und dass sich durch ihn plastische Verschiebung kolloidaler Teilchen gegen ihren viskösen Widerstand als Stauungserrscheinung zu erkennen gibt.

1. Eine Kurve reiner schneller Kontraktion sieht man auf Fig. 2. Am Anfang der Dekreszente wurden 10 gr von dem mit 20 gr belasteten Schreibhebel entfernt: es ergab sich *kein* Schneepflugeffekt: der kleine Haken in der Kurve wurde durch die Manipulation, durch welche die 10 gr entfernt wurden, erzeugt. Nun werden die 10 gr wieder angehängt (total 20 gr), ohne sichtbare Aenderung des Kurvenverlaufs.

2. *Rein tonische Dekreszente*. Wintertiere im frühen Frühjahr befanden sich in einem offenbar geschädigten Zustande. Die Erscheinungen der Muskeltätigkeit erwiesen sich als verzögert und wir haben den Folgen dieses Umstandes unsere Aufmerksamkeit zugewandt. Starke faradische Reizung mit R.A. 6 cm, eine Sekunde lang, hatte in manchen Fällen eine Kontraktion zur Folge, auf welche *keinerlei spontane Erschlaffung folgte*.

Dies zeigte sich, wenn man auf dem Gipfel der Kontraktion eine Pause (Entlastung oder Bremsung) von längerer Zeit machte, z.B. von 15 Minuten, $\frac{1}{2}$ Stunde und in einem Falle von 2 Stunden und 10 Minuten. Öffnen der Bremse oder Neubelastung mit 20 gr hat kaum „freien Fall“ (Dekreszente beinahe in der Richtung der Ordinate¹⁾ zur Folge.

Die Kurve setzt sich hiernach als rein tonische Dehnungskurve fort. Auch zwischendurch eingeschaltete Belastungspausen haben nur einen ganz geringen Einfluss auf den Lauf der Dehnung. An jeder Stelle der Dehnung ist ausgesprochener Schneepflugeffekt zu erzielen. Es fehlt also jede spontane Erschlaffung; die Dehnung ist in jeder Beziehung identisch mit der Ruhedehnung, hervorgerufen durch entsprechendes Gewicht bei sehr tonischem Tiere.

Wir geben ein Beispiel und zwar von demjenigen Fusse, bei dem auf dem Gipfel der Kontraktion 30 Minuten Pause gemacht wurde.

(Reizung während 1 Sek. mit R.A. 6 cm, 14°, 20 gr.)

Höhe der Kontraktion über der Abszisse	6.9 cm
„Freier Fall“ nach Pause, unter Neubelastung	0.2 cm
Totale Ordinatenverminderung nach Dehnung von 5 Min.	2.39 cm
Elevationswinkel dieser Dehnung	20°

Nach 20 Minuten und 50 Sekunden ist die Abszisse noch nicht erreicht (Höhe der Ordinate noch 0.9 cm).

Diese Dehnung ist im üblichen Sinne unserer Tonuslehre „irreversibel“, d.h. es findet keine nennenswerte Wiederverkürzung nach Entlastung statt. Sie beruht auf Ueberwindung des viskösen Widerstandes unter ausgiebiger Verschiebung der Teilchen, durch deren Stauung, wie gesagt, der „Schneepflugeffekt“ entsteht. Daher muss bei der, dieser Dehnung vorhergehenden Verkürzung auch eine Teilchenverschiebung und zwar im Sinne einer Erhöhung des Tonusniveau stattgefunden haben. Nach der Verkürzung sind die kolloidalen Mizellen, als Elemente der Kontraktion, vermutlich nach kürzerer oder längerer Zeit erschlafft; allein dieser Spannungsverlust wird durch den viskösen Widerstand der verkürzten Intermizellarsubstanz maskiert, da sie es ist, die nunmehr die Verkürzung festhält. Leider haben wir es nicht in der Hand diese rein tonische Verkürzung nach Willkür hervorzurufen.

Was geschieht während dieser rein tonischen Kontraktion?

Wenn man einen Streifen plastizierten, nicht vulkanisierten Kautschuk dehnt und sodann entlastet, so tritt eine gewisse Wiederverkürzung („recovery“) auf. Auf dem Gipfel dieser Wiederverkürzung neu belastet, bietet der Streifen dem Gewichte wieder plastischen Widerstand. Daher haben die sich verkürzenden gespannten Mizellen die Intermizellaresubstanz ineinandergeschoben, wie das bei der Holothurie die echten Muskeln mit der plastischen Cutis tun (JORDAN 1914).

In beiden Fällen wird das Tonusniveau erhöht (wenn man bei Kautschuk

¹⁾ Umdrehungsgeschwindigkeit des Kymographions beinahe 9 mm in der Minute.

von Tonus reden will). Offenbar geschieht etwas ähnliches bei der tonischen Kontraktion. Ob bei einer solchen die Intermizellarsubstanz den Mizellen auch aktiv folgen kann, bleibe dahingestellt, wir kommen mit der Annahme passiven Geschobenwerdens, wenigstens für *Helix*, aus, auch wenn diese Annahme nur eine vorläufige Vereinfachung sein sollte.

Reine schnelle Kontraktion muss daher vollkommenes Festwerden der Intermizellarsubstanz zur Voraussetzung haben, da diese den Mizellen bei Kreszente und Dekreszente *ohne Verschiebungen* folgt. (Dieses Festwerden entspricht der Vulkanisierung von Kautschuk.) Hierdurch wird der Muskel zeitweilig zu einem elastischen Strang, was auch für die Uebertragung der Mizellenkontraktion auf die Muskelenden notwendig ist. *Bei unseren Versuchen beruht die tonische Form der Kontraktion daher auf einem Versagen dieses Festwerdens.*

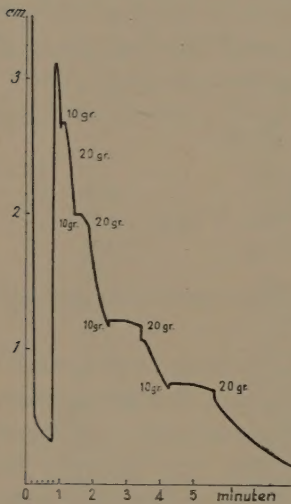


Fig. 3. *Helix pomatia*, Fuss. Gemischte Dekreszente: Gipfel schnelle spontane Erschlaffung ohne Schneeflugeffekt, später tonische Dehnung mit zunehmendem Schneeflugeffekt. Reiz R.A. 12,5 cm, 1 Sek. lang.

3. *Die gemischte Dekreszente* (Fig. 3). Die Erscheinungen, mit denen wir uns beschäftigen, haben gemein, dass es sich um Verzögerung normaler Funktionen handelt, Verzögerung grösseren oder geringeren Grades. Daher kann auch das Festwerden der Intermizellarsubstanz mit *teilweiser* Verzögerung auftreten, und z.B. erst kurz vor dem Gipfel der Kontraktion vollendet sein. Dann muss der erste, oberste Teil der Dekreszente schneller, der folgende Teil langsamer Kontraktion entsprechen und daher die Kontraktion gemischten Typus haben. Die Richtigkeit dieser Erklärung muss sich durch Fehlen von Schneeflugeffekt unterhalb des Gipfels und Auftreten dieser Erscheinung während der tonischen Strecke der Dekres-

zente zeigen. Dies wird durch folgenden Versuch bestätigt. Der Fuss von *Helix pomatia* wird 1 Sek. lang faradisch gereizt, mit R.A. 12,5 cm, Temperatur 14°, Last 20 gr. Es tritt schnelle Kontraktion auf, mit erst schneller Erschlaffung (spitzem Gipfel), die später in tonische Dehnung übergeht.

Nach einer Dekreszente von ungefähr 10 Sekunden ist der Schneeflugeffekt	0	mm
" " " " " "	40	" " " " 1.8 mm
" " " " " "	95	" " " " 5 mm

Spontane Erschlaffung lässt sich im wenig steilen (tonischen) Teil der (hier nur scheinbaren) Dekreszente nicht nachweisen. Dies ergibt sich aus folgenden Versuchen. Wir arbeiten mit einer Reihe von Kontraktionen, mit spitzem Kurvengipfel, deren Dekreszenten bald nach diesem Gipfel

weniger steil werden; immer unter dem Einflusse des Gewichtes von 20 gr. In diesem Teile der Dekreszente nun wird in einigen Fällen durch Verschluss der Bremse eine Pause von z.B. $2\frac{1}{2}$ Minuten eingeschaltet. Nach Oeffnung der Bremse verläuft die Kurve ganz kurze Zeit etwas steiler, wie dies ja auch in der Ruhekurve der Fall ist, allein unmittelbar nimmt die Kurve die ursprüngliche geringe Neigung wieder an.

1. Vor der Pause (erste 1.8 cm der Ordinatenverminderung) verläuft die Kurve mit einem Winkel zur Abszisse von 55° (näher zum Gipfel beträgt der Winkel 77°).
2. Nach der Pause (Gesamtverminderung der Ordinate nach der Pause bis zur Abszisse 2.6 cm):
Die letzte Strecke verläuft mit einem Winkel von 54° .
Die Sehne der ganzen Strecke nach der Pause bildet einen Winkel von 65° .

Ein anderer Versuch: Letzte Strecke vor der Pause 57° .
Strecke nach kurzem „freiem Fall“, nach Pause von 2 Min. 57.5° .

4. *Tonusähnliche Erscheinungen durch verzögerte Erschlaffung auf dem Gipfel der Kontraktion.* Zu dem Komplex der verzögerten Kontraktionserscheinungen gehört auch verzögerte Erschlaffung auf dem Gipfel der Kurve. Sie verursacht folgende Eigentümlichkeiten der Dekreszente, die wir an einem Beispiel verdeutlichen wollen. (Fig. 4.)

Ein „tonischer“ Schneckenfuss, mit einer Last von 20 gr, wird mit R.A. 6 cm faradisch gereizt.

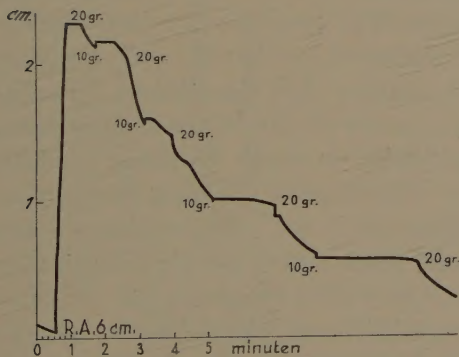


Fig. 4. *Helix pomatia*. Fuss. Gemischte Dekreszente, ausserdem mit verzögerter Erschlaffung auf dem Gipfel. Reiz R.A. 6 cm, 1 Sek. lang.

Auf dem Gipfel verläuft die Kurve nicht in Form einer scharfen Spitze, sondern sie bildet eine horizontale Strecke von 3.9 mm (1 Minute = ungefähr 9 mm); sodann tritt flache Dehnung auf. Nach Dehnung von 20 Sek. entlasten wir auf 10 gr: es tritt Schneepflugeffekt auf mit einer horizontalen Strecke von 5 mm. Hierauf wird wieder mit im Ganzen 20 gr belastet. Die Kurve wird nun steiler, die spontane Erschlaffung

nimmt zu und wenn man nun nach 110 Sekunden (gerechnet vom Beginn der Dehnung am Gipfel) wieder auf 10 gr entlastet, so fehlt eigentlicher Schneepflugeffekt, die Steilheit wird nur etwas vermindert. Es wird aufs Neue mit 20 gr belastet und nach im Ganzen 235 Sek. Dehnung aufs Neue auf 10 gr entlastet. Es ergibt sich Schneepflugeffekt von 10.6 mm.

Aus diesen Versuchen ergibt sich die Tatsache, dass während verzögerter Erschlaffung die Intermizellarsubstanz weitgehend flüssig sein kann. Das

belastende Gewicht dehnt dann den Muskel zeitweilig unter Ueberwindung der Viskosität jener Substanz, welche die Mizellen zu Längsträngen verbindet. Denn die Mizellen widersetzen sich der Dehnung. Wenn später die Erschlaffung schneller geworden ist, beherrscht sie die Dekreszente. Erst in ihrem weiteren Verlaufe wird die Dekreszente wirklich tonisch, nämlich offenbar da, wo die zwischen den kontraktile Strängen liegende ineinandergeschobene Intermizellarsubstanz sich den freien Bewegungen der Stränge widersetzt.

Diese *seitliche* Bremsung (die nach A. V. HILL auch beim quergestreiften Vertebratenmuskel eine grosse Rolle spielt) ist die Grundlage der viskösen Tonuserscheinungen, während der (Kontraktion und der) Erschlaffung. Dies ergibt sich auch aus den folgenden Versuchen.

5. *Der tonische Fuss der Dekreszente schneller Kontraktionen.* Die Abszisse der Kontraktionskurven ist gegeben durch das tonische Niveau von dem aus wir gereizt haben. Reine schnelle Kontraktion geht auf dieses Niveau zurück und die Dekreszente geht hier in die flachere tonische (visköse) Dehnungskurve über. Sehr oft aber ist auch bei solcher schneller Kontraktion dieser Uebergang nicht scharf. Die Kurve biegt schon oberhalb des alten Tonusniveau ab, verliert mehr und mehr an Steilheit, um unscharf in rein tonische Dehnung überzugehen („tonischer Fuss“ der Dekreszente).

Jedem Tonusniveau entspricht eine bestimmte Lagerung der Intermizellarsubstanz. Vor der Reizung wurde das Tonusniveau durch Dehnung erniedrigt. Diese Dehnung bahnte einen Weg innerhalb der Intermizellarsubstanz für die Dekreszente. Daher bestimmt das Niveau nach Dehnung die Abszisse der Kontraktion. Vom Beginn der eigentlichen schnellen Kontraktion ab ist der Muskel fest, er nimmt die Intermizellarsubstanz nicht als Bremse mit in die Höhe. Allein die durch Dehnung erreichte Abszisse ist nur scheinbare Abszisse, da nach jeder Dehnung Spannungen im Muskel bleiben, die bei Entlastung „recovery“ verursachen würden. Offenbar kommt es im ersten Beginn der Kontraktion, ehe die Intermizellarsubstanz fest ist und die eigentliche Verkürzung eingesetzt hat, zu einem Spannungsausgleich und an Stelle der scheinbaren tritt auf etwas höherem Niveau die wirkliche Abszisse. Bis zu ihr ist der Weg für die Dekreszente wirklich gebahnt und daher verläuft sie steil nur bis zum entsprechenden Niveau.

Dass der „tonische Fuss“ die Folge jener Residualspannung ist, lässt sich beweisen. Wenn man nach ausgiebiger passiver Dehnung beide Enden des Muskels (oder eines nicht vulkanisierten plastizierten Kautschukstreifens) eine Zeitlang fixiert, so gleicht sich die Spannung, unter Ueberwindung des viskösen Widerstandes der Intermizellarsubstanz, aus (sogenannte „Relaxation“). Dadurch wird das scheinbare zum wirklichen Tonusniveau. Reizt man einen Schneckenfuss nach hinreichender Relaxation, so *verschwindet der tonische Fuss*, je nach Relaxationsdauer, mehr oder weniger vollkommen.

Dieser Versuch wurde seiner Wichtigkeit wegen oft wiederholt, wir geben in Fig. 5 ein Beispiel und fügen einige Zahlen, Messungen an den betreffenden Kurven, hinzu.

Gemessen wird hauptsächlich die Breite der Kurve, in einem Falle auf der Abszisse, in einem zweiten Falle etwas oberhalb der Abszisse.

	Höhe der Kontraktion	Breite der Kurve auf der Abszisse
1. Ohne Relaxation	9.6 cm	2.4 cm
2. Relaxation (Bremspause) vor der Reizung	10.3 cm	0.89 cm
3. Gleiche Relaxation	8.7 cm	0.8 cm

Breite der Kurve 1.96 cm oberhalb der Abszisse.

1. Ohne vorherige Relaxation	3.6 cm
2. Mit Relaxation	1.15 cm

Relaxation vor der Reizung kann ausschliesslich den Weg für die Dekreszente bahnen. Auf die eigentliche Kontraktion hat sie offenbar keinen Einfluss.

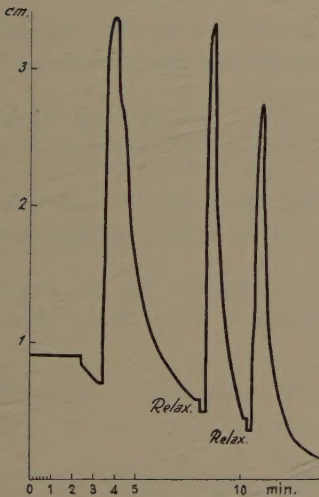


Fig. 5. *Helix pomatia*, Fuss.
„Tonischer Fuss“ der Dekreszente
und seine Vermeidung durch Relaxa-
tion jeweils vor der Reizung. Reiz
R.A. 12,6 cm, 1 Sek. lang.

Wenn dagegen während des Reizversuches das Tonusniveau durch tonische Kontraktion steigt, dann hat vorhergehende Relaxation, wie viele Versuche zeigen, keinen Einfluss auf die Dekreszente, da alsdann von einer irgendwie bestimmten Abszisse für die spontane Erschlaffung keine Rede sein kann.

N. POSTMA hat nunmehr den Einfluss vorhergehender Relaxation auf die durch ihn entdeckte tonische Niveauerhöhung nach Dehnung untersucht und hat gefunden, dass auch diese Form tonischer oder langsamer Kontraktion von der vorherigen Relaxation unabhängig ist.

Die Intermizellarsubstanz, welche die kontraktile Stränge umgibt, kann also in der Tat auf Spontanerschlaffung beruhende Dehnung durch ihren viskosen Widerstand hemmen. Die Bedingungen für diese Hemmung werden nicht durch die Verkürzung selbst erzeugt. Tonus und Kontraktion können unter Umständen zusammen auftreten, sie sind aber prinzipiell unabhängig von einander.

Der Zweck unserer Versuche war, die sehr verschiedenartigen Ursachen zu finden, die an der Gestaltung unserer Kurven beteiligt sind. Wenn man in

zwei verschiedenen Versuchen die gleichen Kurven erhält, so bedeutet das keineswegs, dass man in beiden Fällen die gleiche Ursache aufgedeckt hat. Die Dualität von Tonus und Kontraktion kann nicht bezweifelt werden. Wenn wir auch nicht das Bestehen einer besondern Kontraktion der Intermizellarsubstanz, als Trägerin der Tonusfunktion, ausschliessen wollen, so haben wir doch versucht, auf die überwiegende Bedeutung der Fluiditätsveränderungen bei dieser Substanz für die Erscheinungen des Tonus und seine Wechselwirkung mit der Kontraktion hinzuweisen: Der Tonus von Hohlmuskeln der Vertebraten und von den Muskeln der von uns bearbeiteten Tiere ist eine Funktion des kolloidalen Zustandes dieser Muskeln und seiner Regulierung durch spezifische Centra.

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Physics. — *Researches on heat conduction by rarefied gases. II. The thermal accommodation coefficient of helium, neon, hydrogen, and nitrogen on glass at 70—90° K.* By W. H. KEESOM and G. SCHMIDT. (Abstract of Communication N^o. 245b from the KAMERLINGH ONNES Laboratory at Leiden).

(Communicated at the meeting of October 31, 1936).

The measurements form part of a series of investigations on heat conduction by rarefied gases, viz. helium, neon, hydrogen, and nitrogen at 0° C, 70—90° K, and 14—20° K. The accommodation phenomena are intimately related to those of adsorption.

At 70—90° K adsorption of nitrogen on a glass surface is measurable; hence it can be expected that the accommodation coefficient $a_{1,\infty}$ will be equal to unity. In good agreement with this for N_2 $a_{1,\infty} = 1.02$ is found.

Generally the accommodation coefficient with regard to glass appears to increase with decreasing temperature, to a degree which appears to be connected with the critical temperature.

The table below illustrates this connection, when σ represents the increase of $a_{1,\infty}$ from 0° C to 70—90° K divided by unity minus $a_{1,\infty}$ at 0° C, the latter difference being the maximum increase which would be possible.

He	$(a_{1,\infty})_{0^\circ\text{C}} = 0.336$	$(a_{1,\infty})_{70-90^\circ\text{K}} = 0.383$	$\sigma = 0.07$	$T_{\text{crit.}} = 5.2^\circ\text{K}$
Ne	0.283	0.555	0.38	33.0
H ₂	0.670	0.803	0.40	44.0
N ₂	0.855	1.02	1.00	126.0

On the other hand the critical point can be assumed practically as about the upper limit of the region of measurable adsorption on glass, as follows from the adsorption measurements on He, Ne, and later on H₂ and N₂ on glass.

Combining both facts, we can state the relationship between accommodation-coefficient and adsorption schematically as follows:

1. $T < T_{\text{crit.}}$: adsorption, $a_{1,\infty} = 1$;
2. $T > T_{\text{crit.}}$: no measurable adsorption; at least for a certain temperature range from the critical temperature upwards a rather large increase of $a_{1,\infty}$ can be conceived as an indication of approaching measurable adsorption.

Physics. — *On the scattering of neutrons in matter.* (III). By Prof. L. S. ORNSTEIN. (Communication from the Physical Institute of the University of Utrecht).

(Communicated at the meeting of October 31, 1936).

Let by one or more sources neutrons of an energy ϵ_0 be produced in an infinite layer of protons. We will investigate the distribution law for the energy. Let the production begin at a time $t=0$.

Consider the total number of neutrons of energy ϵ_0 , that is those which have suffered no collision.

For N_0 we get the equation:

$$\frac{dN_0}{dt} = q - a(\epsilon_0) N_0$$

and therefore when at $t=0$ $N_0=0$

$$N_0 = \frac{q}{\alpha_0} (1 - e^{-\alpha(\varepsilon_0)t}).$$

In the stationary state which is approximately reached for times large compared with $\frac{1}{\alpha(\varepsilon_0)}$ we get

$$N_0 = \frac{q}{\alpha_0}.$$

Now we must determine the number of neutrons which suffered only one collision; for this group we must specify the energy between the limits ε and $\varepsilon + d\varepsilon$. Let $N_1(\varepsilon)d\varepsilon$ represent this number. We obtain for this number the differential equation:

$$\frac{dN_1(\varepsilon)}{d\varepsilon} d\varepsilon = \frac{\alpha(\varepsilon_0) v_0(\varepsilon_0) N_0 d\varepsilon}{\varepsilon_0} - N_1(\varepsilon) \alpha(\varepsilon) d\varepsilon.$$

Where $\frac{d\varepsilon}{\varepsilon_0}$ is the probability for the transition ε_0 to ε and $v(\varepsilon_0)$ the probability for the neutron not to be captured at the collision.

The stationary solution is:

$$N_1(\varepsilon) = \frac{1}{\alpha(\varepsilon)} \frac{\alpha(\varepsilon_0) v_0(\varepsilon_0)}{\varepsilon_0} N_0 = \frac{q}{\alpha(\varepsilon)} \frac{v(\varepsilon_0)}{\varepsilon_0}.$$

The total number which suffered only one collision amounts to:

$$N_1 = \frac{q v(\varepsilon_0)}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{\alpha(\varepsilon)}.$$

For the number of neutrons which suffered two collisions and have their energy between ε and $\varepsilon + d\varepsilon$ we get:

$$\frac{dN_2(\varepsilon)}{d\varepsilon} = -\alpha(\varepsilon) N_2(\varepsilon) + \int_{\varepsilon}^{\varepsilon_0} N_1(\varepsilon') \alpha(\varepsilon') v(\varepsilon') \frac{d\varepsilon'}{\varepsilon'}.$$

For the stationary state:

$$\begin{aligned} N_2(\varepsilon) &= \frac{1}{\alpha(\varepsilon)} \int_{\varepsilon}^{\varepsilon_0} N_1(\varepsilon') \alpha(\varepsilon') v(\varepsilon') \frac{d\varepsilon'}{\varepsilon'} \\ &= \frac{1}{\alpha(\varepsilon)} \int_{\varepsilon}^{\varepsilon_0} \frac{v(\varepsilon_0)}{\varepsilon_0} q v(\varepsilon') \frac{d\varepsilon'}{\varepsilon'} \\ &= \frac{1}{\alpha(\varepsilon)} \frac{q v(\varepsilon_0)}{\varepsilon_0} \int_{\varepsilon}^{\varepsilon_0} \frac{v(\varepsilon')}{\varepsilon'} d\varepsilon'. \end{aligned}$$

The total number which suffered only two collisions is:

$$N_2 = \frac{q v(\varepsilon_0)}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{\alpha(\varepsilon)} \int_{\varepsilon}^{\varepsilon_0} \frac{v(\varepsilon')}{\varepsilon'} d\varepsilon'.$$

The number of neutrons which suffered n collisions and possess an energy between ε and $\varepsilon + d\varepsilon$ is $N_n(\varepsilon)d\varepsilon$ with:

$$N_n(\varepsilon) = \frac{q v(\varepsilon_0)}{\varepsilon_0 \alpha(\varepsilon)} \int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi_1) d\xi_1}{\xi_1} \int_{\xi_1}^{\varepsilon_0} \frac{v(\xi_2) d\xi_2}{\xi_2} \dots \int_{\xi_n}^{\varepsilon_0} \frac{v(\xi_n) d\xi_n}{\xi_n}.$$

The total number N_n is given by

$$N_n = \frac{q v(\varepsilon_0)}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{1}{\alpha(\xi_1)} d\xi_1 \int_{\xi_1}^{\varepsilon_0} \frac{v(\xi_2)}{\xi_2} d\xi_2 \dots$$

The total number of neutrons of an energy between ε and $\varepsilon + d\varepsilon$ is given by $\Phi(\varepsilon)d\varepsilon$ where $\Phi(\varepsilon)$ is expressed by the formula

$$\Phi(\varepsilon) = \sum_1^{\infty} N_n(\varepsilon) = \frac{q v_0}{\varepsilon_0 \alpha(\varepsilon)} \left(1 + \int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi_1) d\xi_1}{\xi_1} + \int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi_1) d\xi_1}{\xi_1} \int_{\xi_2}^{\varepsilon_0} \frac{v(\xi_2) d\xi_2}{\xi_2} + \dots \right)$$

The infinite series between the brackets can be calculated. Putting this series $f(\varepsilon)$, we know that $f(\varepsilon_0) = 1$, and it is easily seen that

$$\frac{df(\varepsilon)}{d\varepsilon} = -\frac{v(\varepsilon)}{\varepsilon} f(\varepsilon)$$

so that we get

$$f(\varepsilon) = e^{-\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi} = e^{\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi}.$$

For $\Phi(\varepsilon)$ we thus obtain:

$$\Phi(\varepsilon) = \frac{q v(\varepsilon_0)}{\varepsilon_0 \alpha(\varepsilon)} e^{\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi}.$$

In order to obtain the probability for the energy between ε and $\varepsilon + d\varepsilon$ $P(\varepsilon)d\varepsilon$ we ought to divide by the total number of particles $N_0 + N_1 + N_2 + \dots$

This sum can be obtained in the following way. The first term is $\frac{q}{a_0}$, then results

$$\frac{q v_0}{\varepsilon_0} \left(\int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)} + \int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)} \int_{\varepsilon}^{\varepsilon_0} \frac{v(\varepsilon')}{\varepsilon'} d\varepsilon' + \dots \right)$$

or

$$\frac{q v_0}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)} \left(1 + \int_{\varepsilon}^{\varepsilon_0} \frac{v(\varepsilon')}{\varepsilon'} d\varepsilon' + \dots \right)$$

which is

$$\frac{q v_0}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)} e^{\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi}.$$

Dividing by the total number, we get for $P(\varepsilon)$

$$P(\varepsilon) = \frac{\frac{v(\varepsilon_0)}{\varepsilon_0 a(\varepsilon)} e^{\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi}}{\frac{1}{a(\varepsilon_0)} + \frac{v(\varepsilon_0)}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)} e^{\int_{\varepsilon}^{\varepsilon_0} \frac{v(\xi)}{\xi} d\xi}}.$$

A result which changes the notations analogous to that of FERMI (ZEEMAN-book).

The same result can be obtained in a somewhat different way. If the source q is emitting since the time $-\infty$ and we want to know the number of neutrons at the time t which suffered zero, one, etc. collisions, we get for N_0 :

$$N_0 = q \int_{-\infty}^t d\xi e^{-\alpha(\varepsilon_0)(t-\xi)} = \frac{q}{a(\varepsilon_0)}.$$

For N_1 we get

$$N_1 = q \int_0^{\varepsilon_0} d\varepsilon_1 \int_{-\infty}^t d\xi e^{-\alpha(\varepsilon_0)(t-\xi)} a(\varepsilon_0) dt_1 v(\varepsilon_0) e^{-\alpha(\varepsilon_1)(t-t_1)} \frac{d\varepsilon_1}{\varepsilon_0}$$

or

$$N_1 = \frac{q v(\varepsilon_0)}{\varepsilon_0} \int_0^{\varepsilon_0} \frac{d\varepsilon}{a(\varepsilon)}.$$

In the same way we get

$$N_2 = q \int_0^{\varepsilon_0} d\varepsilon_2 \int_{\varepsilon_3}^{\varepsilon_0} \frac{d\varepsilon_1}{\varepsilon_1} \cdot \\ \cdot \int_{-\infty}^t d\xi e^{-\alpha(\varepsilon_0)(t_1-\xi)} \alpha(\varepsilon_0) v(\varepsilon_0) dt_1 e^{-\alpha(\varepsilon_1)(t_2-t_1)} \alpha(\varepsilon_1) v(\varepsilon_1) dt_2 \cdot e^{-\alpha(\varepsilon_2)(t-t_2)}$$

which integrated gives for N_2 the same expression as we have found with the first method.

Mathematics. — *Generalization of an inequality of KNOPP.* By J. G. VAN DER CORPUT.

(Communicated at the meeting of October 31, 1936).

In this note a_0, a_1, \dots are supposed to be ≥ 0 , not all zero, $q > 0$ and $0 < u \leq 1$ and I write

$$F(u) = \sum_{n=0}^{\infty} u^n \left(a_0 \binom{n}{0}^{q^n} a_1 \binom{n}{1}^{q^{n-1}} \dots a_n \binom{n}{n}^{q^0} \right)^{\frac{1}{(q+1)^n}}.$$

KNOPP¹⁾ has deduced the inequality

$$F(1) < (q+1) \sum_{n=0}^{\infty} a_n, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

provided that the last series converges; for any fixed q the constant $q+1$ is the best possible.

As $(q+1)v^q - qv^{q+1}$ is in the interval $0 \leq v \leq 1$ a continuous monotonic increasing function of v , assuming for $v=0$ and $v=1$ the values 0 and 1, there is one and only one v ($0 \leq v \leq 1$) such that

$$(q+1)v^q - qv^{q+1} = u^{q+1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In this paper I will prove:

$$F(u) \leq \frac{(q+1)v^q}{u^{q+1}} \sum_{n=0}^{\infty} a_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

provided that the last series converges; for any fixed u and q the constant $\frac{(q+1)v^q}{u^{q+1}}$ is the best possible.

¹⁾ K. KNOPP, Ueber Reihen mit positiven Gliedern (Zweite Mitteilung), Journal Lond. Math. Soc. 5 13—21, (1930).

For $u=1$ relation (3) is always valid with the sign of inequality. If $0 < u < 1$, the sign of equality in (3) holds when and only when

$$a_n = p \frac{v^{(q+1)n}}{u^{(q+1)n}} \quad (p > 0).$$

In the special case $q=1$ relation (2) takes the form $2v - v^2 = u^2$,

$$v = 1 - \sqrt{1-u^2} = \frac{u^2}{1 + \sqrt{1-u^2}} \text{ and } \frac{(q+1)v^q}{u^{q+1}} = \frac{2}{1 + \sqrt{1-u^2}},$$

hence

$$\sum_{n=0}^{\infty} u^n \left(a_0 \binom{n}{0} a_1 \binom{n}{1} \dots a_n \binom{n}{n} \right)^{\frac{1}{2^n}} \leq \frac{2}{1 + \sqrt{1-u^2}} \sum_{n=0}^{\infty} a_n, \quad (4)$$

and the constant $\frac{2}{1 + \sqrt{1-u^2}}$ is the best possible. If $u=1$ relation (4) is true with the sign of inequality. If $u < 1$ the sign of inequality holds when and only when

$$a_n = p \left(\frac{u}{1 + \sqrt{1-u^2}} \right)^{2^n} \quad (p > 0).$$

Relation (4) involves many inequalities, for instance integration gives

$$\sum_{n=0}^{\infty} \frac{1}{n+1} \left(a_0 \binom{n}{0} a_1 \binom{n}{1} \dots a_n \binom{n}{n} \right)^{\frac{1}{2^n}} < (\pi-2) \sum_{n=0}^{\infty} a_n,$$

but $\pi-2$ is not the best possible constant.

The proof runs as follows:

Putting $w = \frac{v}{u}$ we obtain

$$\begin{aligned} F(u) &= \sum_{n=0}^{\infty} u^n \prod_{m=0}^n a_m \binom{n}{m} \frac{q^{n-m}}{(q+1)^n} \\ &= \sum_{n=0}^{\infty} u^n w^{-nq} \prod_{m=0}^n (a_m w^{(n-m)(q+1)}) \binom{n}{m} \frac{q^{n-m}}{(q+1)^n} \end{aligned}$$

in virtue of

$$\sum_{m=0}^n (n-m) \binom{n}{m} q^{n-m} = nq(q+1)^{n-1}.$$

The theorem of the arithmetic and geometric means gives

$$F(u) < \sum_{n=0}^{\infty} u^n w^{-nq} \sum_{m=0}^n \binom{n}{m} \frac{q^{n-m}}{(q+1)^n} a_m w^{(n-m)(q+1)} \dots \quad (5)$$

unless the $n+1$ numbers $a_m w^{(n-m)(q+1)}$ ($m=0, 1, \dots, n$) are equal to one another; in this last case the sign of inequality has to be replaced by the sign of equality. The right side of (5) is

$$\begin{aligned} \sum_{m=0}^{\infty} u^m w^{-mq} (q+1)^{-m} a_m \sum_{n=m}^{\infty} \binom{n}{m} \left(\frac{quw}{q+1} \right)^{n-m} \\ = \sum_{m=0}^{\infty} u^m w^{-mq} (q+1)^{-m} a_m \left(1 - \frac{quw}{q+1} \right)^{-m-1} \end{aligned}$$

for (2) implies

$$1 - \frac{quw}{q+1} = 1 - \frac{qv}{q+1} = \frac{u^{q+1}}{v^q} > 0,$$

hence $\frac{quw}{q+1} < 1$. Consequently

$$F(u) < \frac{(q+1)w^q}{u} \sum_{m=0}^{\infty} a_m = \frac{(q+1)v^q}{u^{q+1}} \sum_{n=0}^{\infty} a_n$$

unless $a_m = p w^{(q+1)m}$ ($p > 0$); in this last case the sign of inequality is to be replaced by the sign of equality. If $u < 1$, the left side of (2) has for $v = u$ the value

$$(q+1)u^q - qu^{q+1} > (q+1)u^{q+1} - qu^{q+1} = u^{q+1},$$

so that $v < u$, $w < 1$ and the case $a_m = p w^{(q+1)m}$ is possible. If $u = 1$, we have $w = 1$, the case $a_m = p w^{(q+1)m}$ is excluded and (1) is true; if we choose $a_m = s^{m(q+1)}$ ($0 < s < 1$),

$$F(1) = \sum_{n=0}^{\infty} s^n = \frac{1}{1-s}$$

and the right side of (1) is

$$(q+1) \sum_{n=0}^{\infty} s^{n(q+1)} = \frac{q+1}{1-s^{q+1}}.$$

If $s \rightarrow 1$

$$\frac{1-s^{q+1}}{(q+1)(1-s)} \rightarrow 1;$$

hence it follows that $q+1$ is in (1) the best possible constant.

Mathematics. — On KUMMER's solutions of the hypergeometric differential equation. By J. G. VAN DER CORPUT.

(Communicated at the meeting of October 31, 1936).

Consider the analytic function

$$\psi(\alpha, \beta, \gamma; z) = \frac{z^{\alpha+1/6} (1-z)^{\beta+1/6}}{\Gamma(1+2\alpha)} F\left(\frac{1}{2} + \alpha + \beta + \gamma, \frac{1}{2} + \alpha + \beta - \gamma; 1 + 2\alpha; z\right), \quad (1)$$

where $|\arg z| < \pi$ and $|\arg (1-z)| < \pi$; $\pm 2\alpha$, $\pm 2\beta$ and $\pm 2\gamma$ are supposed not to be integers or zero.

This function is unchanged when γ is replaced by $-\gamma$.

Theorem 1: The function $\psi(\alpha, \beta, \gamma; z)$ is a solution of the homogeneous linear differential equation of the second order ¹⁾

$$\frac{d^2 w}{dz^2} + p(z) \frac{dw}{dz} + q(z)w = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which has every point except 0, 1 and ∞ as an ordinary point, these three points being regular points with exponents $\frac{1}{6} \pm \alpha$, $\frac{1}{6} \pm \beta$ and $\frac{1}{6} \pm \gamma$. This equation has also the solution $\psi(-\alpha, \beta, \gamma; z)$ and any solution has the form

$$A \psi(\alpha, \beta, \gamma; z) + B \psi(-\alpha, \beta, \gamma; z), \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where A and B are constants.

The Function F occurring in (1) satisfies the homogeneous linear differential equation of the second order, which has every point except 0, 1 and ∞ as an ordinary point, these three points being regular points; the exponents in 0 are 0 and $1 - (1 + 2\alpha) = -2\alpha$; those in 1 are 0 and

$$(1 + 2\alpha) - \left(\frac{1}{2} + \alpha + \beta + \gamma\right) - \left(\frac{1}{2} + \alpha + \beta - \gamma\right) = -2\beta$$

and finally in ∞

$$\frac{1}{2} + \alpha + \beta + \gamma \quad \text{and} \quad \frac{1}{2} + \alpha + \beta - \gamma.$$

Consequently the function ψ defined by (1) is a solution of the homo-

¹⁾ It is not necessary to observe that in this equation

$$p(z) = \frac{4z-2}{3z(z-1)}$$

and

$$q(z) = \frac{1 - (1 + 36\beta^2)z + z^2}{36z^2(z-1)^2} + \frac{\alpha^2}{z^2(z-1)} - \frac{\gamma^2}{z(z-1)}.$$

geneous linear differential equation of the second order with the exponents

$$(0 \text{ or } -2\alpha) + \alpha + \frac{1}{6} = \frac{1}{6} \pm \alpha \quad \text{in } 0,$$

$$(0 \text{ or } -2\beta) + \beta + \frac{1}{6} = \frac{1}{6} \pm \beta \quad \text{in } 1$$

and

$$\frac{1}{2} + \alpha + \beta \pm \gamma - (\alpha + \beta + \frac{1}{6}) = \frac{1}{6} \pm \gamma \quad \text{in } \infty,$$

i.e. equation (2).

This equation, that is unchanged when α is replaced by $-\alpha$, has accordingly also the solution $\psi(-\alpha, \beta, \gamma; z)$. These two solutions are not multiples of each other and form so a fundamental system, so that any solution of (2) has the form (3).

Theorem 2: Equation (2) has the solutions

$$\left\{ \begin{array}{l} \psi(\pm\alpha, \pm\beta, \gamma; z), \psi(\pm\beta, \pm\alpha, \gamma; 1-z), \psi\left(\pm\gamma, \pm\beta, \alpha; \frac{1}{z}\right), \\ \psi\left(\pm\alpha, \pm\gamma, \beta, \frac{z}{z-1}\right), \psi\left(\pm\beta, \pm\gamma, \alpha, \frac{z-1}{z}\right), \psi\left(\pm\gamma, \pm\alpha, \beta, \frac{1}{1-z}\right). \end{array} \right.$$

For any of the transformations $Z = z, 1-z, \frac{1}{z}, \frac{z}{z-1}, \frac{z-1}{z}$ and $\frac{1}{1-z}$ transforms the system $(0, 1, \infty)$ in itself. These 24 solutions give by (1) KUMMER's 24 solutions of the hypergeometric differential equation.

Theorem 3: The four functions $\psi(\alpha, \pm\beta, \gamma, z)$ and $\left(\alpha, \pm\gamma, \beta, \frac{z}{z-1}\right)$ are equal to one another throughout a suitable chosen domain.

For any of this four functions is a solution of (2) and has therefore form (3). Investigation of the behaviour of the functions in the vicinity of 0 gives $B=0$ and $A=1$.

In theorem 3 we may replace z and $\frac{z}{z-1}$ by $1-z$ and $\frac{z-1}{z}$, or by $\frac{1}{z}$ and $\frac{1}{1-z}$.

In this manner we can group the 24 functions mentioned in theorem 2 (accordingly also KUMMER's 24 solutions of the hypergeometric differential equation) into six sets of four, such that the members of the same set are constant multiples of one another throughout a suitably chosen domain.

Theorem 4: If $|\arg z| < \pi$ and $|\arg(1-z)| < \pi$, we have

$$\frac{1}{\pi} (\sin 2\pi\alpha) \psi(\beta, \alpha, \gamma; 1-z) = \frac{\psi(-\alpha, \beta, \gamma; z)}{\Gamma(\frac{1}{2} + \alpha + \beta + \gamma) \Gamma(\frac{1}{2} + \alpha + \beta - \gamma)} - \frac{\psi(\alpha, \beta, \gamma; z)}{\Gamma(\frac{1}{2} - \alpha + \beta + \gamma) \Gamma(\frac{1}{2} - \alpha + \beta - \gamma)}.$$

The left side is a solution of (2), consequently

$$\frac{1}{\pi} (\sin 2\pi\alpha) \psi(\beta, \alpha, \gamma; 1-z) = A \psi(\alpha, \beta, \gamma; z) + B \psi(-\alpha, \beta, \gamma; z) \quad (4)$$

If $\Re \alpha < 0$, $0 < z < 1$ and $z \rightarrow 0$, we obtain by (1)

$$z^{-\alpha-1/2} \psi(\beta, \alpha, \gamma; 1-z) \rightarrow \frac{F(\frac{1}{2} + \alpha + \beta + \gamma, \frac{1}{2} + \alpha + \beta - \gamma; 1 + 2\beta; 1)}{\Gamma(1 + 2\beta)}$$

$$= \frac{\Gamma(-2\alpha)}{\Gamma(\frac{1}{2} - \alpha + \beta - \gamma) \Gamma(\frac{1}{2} - \alpha + \beta + \gamma)};$$

$\frac{1}{\pi} \sin 2\pi\alpha$ times this result is $\frac{A}{\Gamma(1 + 2\alpha)}$, hence if $\Re \alpha < 0$

$$A = \frac{1}{\pi} \sin 2\pi\alpha \cdot \frac{\Gamma(1 + 2\alpha) \Gamma(-2\alpha)}{\Gamma(\frac{1}{2} - \alpha + \beta + \gamma) \Gamma(\frac{1}{2} - \alpha + \beta - \gamma)}$$

$$= \frac{-1}{\Gamma(\frac{1}{2} - \alpha + \beta + \gamma) \Gamma(\frac{1}{2} - \alpha + \beta - \gamma)}.$$

Similarly if $\Re \alpha > 0$

$$B = \frac{1}{\Gamma(\frac{1}{2} + \alpha + \beta + \gamma) \Gamma(\frac{1}{2} + \alpha + \beta - \gamma)}.$$

From (4) and the relation, obtained by differentiation of (4) to z , we obtain A and B as fractions whose numerators and denominators are continuous functions of α . The values found for A and B are therefore valid if $\Re \alpha = 0$, consequently (by the principle of analytic continuation) for any α .

In this theorem I replace β, γ, z by $\gamma, \beta, \frac{z}{z-1}$ and find by means of theorem 3:

Theorem 5: If $|\arg(-z)| < \pi$ and $|\arg(1-z)| < \pi$, we have

$$\frac{1}{\pi} (\sin 2\pi\alpha) \psi\left(\gamma, \beta, \alpha; \frac{1}{z}\right) = \frac{\psi(-\alpha, \beta, \gamma; z)}{\Gamma(\frac{1}{2} + \alpha + \beta + \gamma) \Gamma(\frac{1}{2} + \alpha - \beta + \gamma)} - \frac{\psi(\alpha, \beta, \gamma; z)}{\Gamma(\frac{1}{2} - \alpha + \beta + \gamma) \Gamma(\frac{1}{2} - \alpha - \beta + \gamma)}.$$

So I obtain, almost without any calculation, the well known linear relations between KUMMER's 24 solutions of the hypergeometric differential equation, for instance

$$\begin{aligned} \frac{\Gamma(a) \Gamma(b)}{\Gamma(c)} F(a, b; c; z) &= \frac{\Gamma(a) \Gamma(a-b)}{\Gamma(a-c)} (-z)^{-a} F(a, 1-c+a; 1-b+a; z^{-1}) \\ &+ \frac{\Gamma(b) \Gamma(b-a)}{\Gamma(b-c)} (-z)^{-b} F(b, 1-c+b; 1-a+b; z^{-1}) \end{aligned}$$

where $|\arg(-z)| < \pi$, and

$$\frac{F(a, b; c; z)}{\Gamma(c) \Gamma(c-a-b)} = \frac{F(a, b; a+b-c+1; 1-z)}{\Gamma(c-a) \Gamma(c-b)} \\ + (1-z)^{c-a-b} \frac{F(c-a, c-b; c-a-b+1; 1-z)}{\Gamma(a) \Gamma(b)}$$

where $|\arg(-z)| < \pi$ and $|\arg(1-z)| < \pi$; the first result has to be modified when $a-b$ is an integer or zero, the second when $c-a-b$ is an integer or zero.

Physics. — *Ueber die konforminvariante Gestalt der relativistischen Bewegungsgleichungen.* Von J. A. SCHOUTEN und J. HAANTJES.

(Communicated at the meeting of October 31, 1936).

1. Einleitung.

In einer früheren Arbeit ¹⁾ haben wir gezeigt, dass sich die MAXWELLSchen Gleichungen und die Impulsenergiegleichungen konforminvariant schreiben lassen. In dieser Arbeit wird gezeigt, dass sich auch die relativistischen Bewegungsgleichungen geladener Teilchen in eine konforminvariante Form bringen lassen, vorausgesetzt dass man die Masse so mit transformiert, dass das Produkt von Masse und Länge invariant bleibt. Es spielt dann $\frac{h}{c}$ (Dim. $[ML]$) eine ähnliche Rolle wie c in der gewöhnlichen Relativitätstheorie. Statt der Ruhmasse $\overset{\circ}{m}$ kommt eine andere Invariante, die Konformmasse $\overset{c}{m} = \overset{\circ}{m} (-g)^{1/8}$, die die Dimension $[ML]$ hat. Wir geben hier nur die einfachen mathematischen Tatsachen und vermeiden physikalische Spekulationen.

Wir erinnern kurz an die früher erhaltenen Resultate. In einer Raumzeitwelt mit einer konformen Metrik gibt es keinen Fundamentaltensor g_{ih} , dagegen eine Tensordichte $\mathfrak{G}_{ih} = g_{ih} (-g)^{-1/4}$ ($g = \text{Det. } g_{ih}$) vom Gewicht $-1/2$. Es gibt kein Linienelement $d\tau$, dagegen ein konformes (dimensionsloses) Linienelement $d\hat{s}$, definiert durch:

$$(d\hat{s})^2 = \mathfrak{G}_{ih} d\xi^i d\xi^h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

¹⁾ Ueber die konforminvariante Gestalt der MAXWELLSchen Gleichungen und der elektromagnetischen Impulsenergiegleichungen, *Physica I* (1934), 869–872.

Die Ladung eines vierdimensionalen Volums $d\omega$, die selbst konform-invariant ist, legt mittels der Gleichung

$$de d\mathfrak{s} = \varrho d\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

eine Ladungsdichte vom Gewicht $+\frac{3}{4}$ (Dim.: $M^{1/2} L^{3/2} T^{-1}$) fest und aus dieser ergibt sich die Stromvektordichte vom Gewicht $+1$

$$\mathfrak{s}^h = \varrho \frac{d\xi^h}{d\mathfrak{s}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Es gelten die Gleichungen

$$\left. \begin{aligned} F_{ji} &= 2\partial_{[j} \varphi_{i]} ; \partial_j = \frac{\partial}{\partial \xi^j} ; (\text{elektromagn. Feld}) \\ \partial_{[j} F_{ih]} &= 0 \\ \mathfrak{F}^{hi} &= \mathfrak{G}^{hi} \mathfrak{G}^{ij} F_{ij} \\ \mathfrak{s}^h &= -\partial_j \mathfrak{F}^{jh} \\ \partial_j \mathfrak{s}^j &= 0 \\ \mathfrak{S}^h_{\cdot i} &= -\mathfrak{F}^{hj} F_{ij} + \frac{1}{4} F_{ij} \mathfrak{F}^{ij} A_i^h \quad (\text{Impulsenergiesordichte}) \\ -\nabla_h \mathfrak{S}^h_{\cdot i} &= \mathfrak{s}^j F_{ji} = \mathfrak{f}_i \quad (\text{Kraftvektordichte}) \end{aligned} \right\} \quad (4)$$

Bis auf die letzte Gleichung treten keine kovarianten Differentiationen auf. Die ersten sechs Gleichungen sind also von jeder Wahl einer symmetrischen Uebertragung unabhängig, in dem Sinne, dass bei jeder solchen Wahl ∂_j durch ∇_j ersetzt werden darf. In der oben zitierten Arbeit haben wir gezeigt, dass die letzte Gleichung gilt für jede Uebertragung, für die $\nabla_j \mathfrak{G}^{hi} = 0$ ist.

Bekanntlich legt \mathfrak{G}_{ih} allein keine Uebertragung, also auch keine geodätischen Linien fest ausser den *geodätischen Nulllinien*, die von jeder Wahl der Uebertragung unabhängig sind (Beweis im nächsten Abschnitt). Wenn also bestimmte Weltlinien resultieren sollen, so muss irgend etwas hinzugefügt werden, und es ist bekannt, dass die Forderung der Linearität der Uebertragung zwangsläufig zu einer WEYLSchen Uebertragung führt.

2. Parallelverschiebung von Vektordichten bei einer WEYLSchen Uebertragung in X_4 .

Sind Γ_{ji}^h die Parameter einer linearen Uebertragung, so ist das kovariante Differentialquotient einer Dichte η^* vom Gewicht \mathfrak{k} gegeben durch die Gleichung

$$\nabla_j \eta^h = \partial_j \eta^h + \Gamma_{ji}^h \eta^i - \mathfrak{k} \Gamma_{ji}^i \eta^h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Bei einer WEYLSchen Uebertragung ist

$$\Gamma_{ji}^h = \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} + \frac{1}{2} (Q_j A_i^h + Q_i A_j^h - \mathfrak{G}^{hk} \mathfrak{G}_{ij} Q_k), \quad . \quad . \quad . \quad (6)$$

wo Q_i ein Vektor ist, der sich bei der konformen Transformation („Umeichung“)

$$g'_{ih} = \sigma g_{ih} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

folgendermassen transformiert

$$Q'_j = Q_j - \partial_j \log \sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Da $\left\{ \begin{matrix} i \\ ji \end{matrix} \right\} = \frac{1}{2} \partial_j \log(-g)$ ist, lautet die Differentiationsgleichung einer Vektordichte bei einer WEYLSchen Uebertragung

$$\begin{aligned} \nabla_j \eta^h = \partial_j \eta^h + \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} \eta^i - \frac{1}{2} \mathfrak{f} \eta^h \partial_j \log(-g) + \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} \cdot \quad . \quad . \quad (9) \\ + \frac{1}{2} ((1-4\mathfrak{f}) Q_j A_i^h + Q_i A_j^h - \mathfrak{G}^{hk} \mathfrak{G}_{ij} Q_k) \eta^i \end{aligned}$$

Man leitet aus dieser Gleichung leicht ab, dass $\nabla_j \mathfrak{G}_{ih} = 0$ ist (unabhängig von der Wahl von Q_i).

Da $\frac{d\xi^h}{d\mathfrak{s}}$ das Gewicht $\frac{1}{4}$ hat, lautet die Gleichung einer geodätischen Linie, die nicht Nulllinie ist

$$\begin{aligned} \frac{\partial}{d\mathfrak{s}} \frac{d\xi^h}{d\mathfrak{s}} = \frac{d\xi^j}{d\mathfrak{s}} \nabla_j \frac{d\xi^h}{d\mathfrak{s}} = \frac{d\xi^j}{d\mathfrak{s}} \partial_j \frac{d\xi^h}{d\mathfrak{s}} + \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} \frac{d\xi^i}{d\mathfrak{s}} \frac{d\xi^j}{d\mathfrak{s}} - \left. \begin{aligned} & - \frac{1}{8} \frac{d\xi^h}{d\mathfrak{s}} \frac{d\xi^j}{d\mathfrak{s}} \partial_j \log(-g) + \frac{1}{2} (Q_i A_j^h - \mathfrak{G}^{hk} \mathfrak{G}_{ij} Q_k) \frac{d\xi^i}{d\mathfrak{s}} \frac{d\xi^j}{d\mathfrak{s}} = 0. \end{aligned} \right\} \quad (10) \end{aligned}$$

Ist die geodätische Linie aber Nulllinie, so ist $d\mathfrak{s} = 0$. Man kann aber einen beliebigen skalaren Parameter z auf die Linie legen. Die Gleichung lautet dann

$$\begin{aligned} \frac{\partial}{dz} \frac{d\xi^h}{dz} = \frac{d\xi^j}{dz} \partial_j \frac{d\xi^h}{dz} + \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} \frac{d\xi^i}{dz} \frac{d\xi^j}{dz} + \frac{1}{2} (Q_i A_j^h + Q_j A_i^h - \left. \begin{aligned} & - \mathfrak{G}^{hk} \mathfrak{G}_{ij} Q_k) \frac{d\xi^i}{dz} \frac{d\xi^j}{dz} :: \frac{d\xi^h}{dz} \end{aligned} \right\} \quad (11) \end{aligned}$$

($::$ = proportional zu) und es folgt also, dass die geodätischen Nulllinien von der Wahl von Q_i unabhängig sind.

Die Uebertragung heisst pseudo-WEYLSch, wenn der Vektor Q_i sich auf Null umeichen lässt, d.h. wenn er Gradientvektor ist. Ist diese Umeichung geschehen und hat man damit den Fundamentaltensor eingeführt i.b. auf welchen die Uebertragung eine RIEMANNsche ist, so geht (9) über in

$$\nabla_j \eta^h = \partial_j \eta^h + \left\{ \begin{matrix} h \\ j i \end{matrix} \right\} \eta^i - \frac{1}{2} \eta^h \partial_j \log(-g) \quad . \quad . \quad (12)$$

Aus den I_{ji}^h einer allgemeinen WEYLSchen Uebertragung kann man in bekannter Weise den Krümmungsaffinor R_{kji}^h , daraus die Grösse $R_{ji} = R_{hji}^h$ und schliesslich die Dichte $\mathfrak{R} = R_{ji} \mathfrak{G}^{ji}$ vom Gewicht $\frac{1}{2}$ bilden. Diese Dichte legt einen Fundamentaltensor $\varepsilon^2 \hat{g}_{ih} = \mathfrak{G}_{ih} \mathfrak{R}$ und damit ein absolutes (kosmologisch bestimmtes) Mass fest. Die konstante ε (Dimension $[L^{-1}]$) ist so gewählt, dass die gewöhnlichen Masseinheiten resultieren, also ist ε sehr klein, da in materiefreien Gebieten \mathfrak{R} jedenfalls sehr klein ist ¹⁾.

Führen wir für den Skalar $\mathfrak{R}(-g)^{-\frac{1}{2}}$, der bei Umeichung den Faktor σ^{-1} bekommt, die Bezeichnung $\varepsilon^2 S$ ein, so ist bei Umeichung

$$\partial_j \log S' = \partial_j \log S - \partial_j \log \sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

und daraus geht hervor dass

$$Q_j = P_j + \partial_j \log S \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

ist, wo P_j ein bei Umeichung invarianter Vektor ist. Bei Umeichung auf absolutes Mass wird $S=1$ und $Q_j=P_j$.

3. Ableitung der konforminvarianten Gestalt der Bewegungsgleichungen.

Die klassisch-relativistischen Bewegungsgleichungen für einen Punkt mit Ruhmasse $\overset{\circ}{m}$ und Ladung e im leeren Raum lauten

$$\overset{\circ}{m} \left(\frac{d^2 \xi^h}{d\tau^2} + \left\{ \begin{matrix} h \\ j i \end{matrix} \right\} \frac{d\xi^j}{d\tau} \frac{d\xi^i}{d\tau} \right) = \frac{e}{c} \frac{d\xi^i}{d\tau} F_{ij} g^{hj} \quad . \quad . \quad . \quad (15)$$

Führen wir statt $d\tau$ das konforminvariante Linienelement $d\hat{s} = (-g)^{-1/2} c d\tau$ ein, so geht die Gleichung über in:

$$\frac{d^2 \xi^h}{d\hat{s}^2} + \left\{ \begin{matrix} h \\ j i \end{matrix} \right\} \frac{d\xi^j}{d\hat{s}} \frac{d\xi^i}{d\hat{s}} - \frac{1}{8} \frac{d\xi^h}{d\hat{s}} \frac{d\xi^j}{d\hat{s}} \partial_j \log(-g) = \frac{e}{\overset{\circ}{m} c^2} (-g)^{-1/2} \frac{d\xi^i}{d\hat{s}} F_{ij} \mathfrak{G}^{hj} \quad (16)$$

¹⁾ H. WEYL, R. Z. M.; vierte Auflage, S. 269.

Da $\frac{e}{\dot{m} c^2} (-g)^{-1/8} F_{ij}$ dimensionslos ist und \mathbb{G}^{hj} und $\frac{d\xi^i}{d\mathfrak{s}}$ konforminvariant sind, ist die rechte Seite konforminvariant. Vergleichung mit (10) lehrt, dass die linke Seite genau gleich $\frac{\delta}{d\mathfrak{s}} \frac{d\xi^h}{d\mathfrak{s}}$ ist für eine pseudo-WEYLSche Uebertragung deren Q_i gerade zufällig auf Null reduziert ist. Die für beliebige konforme Transformationen invariante Form der Gleichung lautet also

$$\left. \begin{aligned} \frac{\delta}{d\mathfrak{s}} \frac{d\xi^h}{d\mathfrak{s}} &= \frac{d^2 \xi^h}{d\mathfrak{s}^2} + \left\{ \begin{matrix} h \\ j i \end{matrix} \right\} \frac{d\xi^j}{d\mathfrak{s}} \frac{d\xi^i}{d\mathfrak{s}} - \frac{1}{8} \frac{d\xi^h}{d\mathfrak{s}} \frac{d\xi^i}{d\mathfrak{s}} \partial_i \log(-g) + \\ + \frac{1}{2} (A_j^h P_i - \mathbb{G}^{hk} \mathbb{G}_{ij} P_k) \frac{d\xi^j}{d\mathfrak{s}} \frac{d\xi^i}{d\mathfrak{s}} + \frac{1}{2} (A_j^h \partial_i \log S - \mathbb{G}^{hk} \mathbb{G}_{ij} \partial_k \log S) \frac{d\xi^j}{d\mathfrak{s}} \frac{d\xi^i}{d\mathfrak{s}} &= \\ &= \frac{e}{\dot{m} c^2} (-g)^{-1/8} \frac{d\xi^i}{d\mathfrak{s}} F_{ij} \mathbb{G}^{hj}, \end{aligned} \right\} \quad (17)$$

wo P_i vorläufig noch als Gradientvektor gedacht ist. Führt man wieder den Parameter τ ein, so lautet die Gleichung

$$\left. \begin{aligned} \frac{d^2 \xi^h}{d\tau^2} + \left\{ \begin{matrix} h \\ j i \end{matrix} \right\} \frac{d\xi^j}{d\tau} \frac{d\xi^i}{d\tau} + \frac{1}{2} (A_j^h Q_i - \mathbb{G}^{hk} \mathbb{G}_{ij} Q_k) \frac{d\xi^j}{d\tau} \frac{d\xi^i}{d\tau} &= \\ &= \frac{e}{\dot{m} c} \frac{d\xi^i}{d\tau} F_{ij} g^{jh}. \end{aligned} \right\} \quad (18)$$

Da die MAXWELLSchen Gleichungen konforminvariant sind, folgt, dass sowohl e als F_{ij} konforminvariant sein müssen. Da auch die Konforminvarianz von c ausser Frage ist, kann $\frac{e}{\dot{m} c^2} (-g)^{-1/8} F_{ij}$ nur dann Dimensionslos sein, wenn \dot{m} bei der Transformation (7) einen Faktor $\sigma^{-\frac{1}{2}}$ bekommt. Rechts in (17) kommt dann im Nenner die konforminvariante Masse $\dot{m} = \dot{m} (-g)^{1/8}$ mit der Dimension $[ML]$. Zu dieser Masse gehört die konforminvariante Massendichte $\dot{\mu}$, definiert durch die Gleichung $c d\dot{m} d\tau = d\dot{m} d\mathfrak{s} = \dot{\mu} d\omega$, die ebenfalls die Dimension $[ML]$ hat. $\dot{\mu} c$ ist die konforminvariante Aktionsdichte. Transformation der Längen mit $\sigma^{\frac{1}{2}}$ muss also Transformation der Massen mit $\sigma^{-\frac{1}{2}}$ mit sich bringen. Dabei bleibt $\frac{h}{c}$ invariant und diese Konstante spielt also beim Uebergang zur konformen Relativitätstheorie eine ähnliche Rolle wie c beim Uebergang zur gewöhnlichen. Da die Dimensionen von e und F_{ij} beide $[M^{1/2} L^{3/2} T^{-1}] = [M^{1/2} L^{1/2} \cdot L T^{-1}]$ sind, führt übrigens auch schon die Forderung der Konforminvarianz dieser Grössen zur Invarianz von $[ML]$.

Bekanntlich wird in der WEYLSchen Theorie Q_i von vornherein mit dem

unbestimmten elektromagnetischen Potentialvektor φ_i identifiziert. Abgesehen davon, dass dazu die an sich völlig freie Transformation von φ_i in einer physikalisch nicht recht begründeten Weise mit der Umeichung verknüpft werden muss, entsteht in dieser Weise eine WEYLSche Uebertragung, die jedenfalls für die Weltlinien freier Teilchen keine Bedeutung haben kann. Denn setzt man in (17) $P_i + \partial_i \log S$ dem Potentialvektor gleich, so ergeben sich Weltlinien, die für keine Festlegung des Potentialvektors mit den richtigen Weltlinien übereinstimmen können. Wir wollen dagegen fordern, dass die Gleichung (17), kurz geschrieben

$$\frac{\delta}{d\mathfrak{s}} \frac{d\xi^h}{d\mathfrak{s}} = \frac{e}{m c^2} \frac{d\xi^i}{d\mathfrak{s}} F_{ij} \mathfrak{G}^{hj} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

mit hinreichender Genauigkeit die richtigen experimentell genügend gesicherten Weltlinien ergibt, wenn das kosmologisch bestimmte natürliche Mass zu grunde gelegt wird. Daraus geht hervor, dass der bei Umeichung invariante Vektor P_i , ob Gradientvektor oder nicht, jedenfalls in materiefreien Gebiete sehr klein ist den anderen in der Gleichung vorkommenden Grössen gegenüber. Denn bei Umeichung auf natürliches Mass verschwindet der Term mit S und muss die Gleichung bis auf eine kleine mit den Messresultaten verträgliche Abweichung in (15) übergehen. Es wäre nicht unmöglich dass P_i u. a. einen Term enthält von der Form

$\alpha \partial_i \log \frac{c}{\mathfrak{R}}^{1/2}$, wo α irgend eine Konstante darstellt. Dies ergäbe eine

eventuell experimentell zugängliche Abweichung der Weltlinien abhängig von der konforminvarianten Aktionsdichte.

4. Die konforminvariante Gestalt der DIRACschen Gleichung.

Es ist in letzter Zeit von verschiedenen Autoren versucht worden eine konforminvariante DIRACsche Gleichung auf zu stellen¹⁾.

DIRAC kommt zu dem Resultat, dass es keinen einfachen Weg gibt zu einer solchen Gleichung zu gelangen²⁾. Unter konforminvariant wird dabei verstanden nur abhängig von \mathfrak{G}_{ih} , nicht noch von irgendeinem Felde Q_i . Wir lassen die letzte Beschränkung fallen, da es ja nun tatsächlich in der Natur mal Weltlinien, also irgend eine Art von Uebertragung, zu geben scheint und betrachten die DIRACsche Gleichung in der Form

$$\left(\frac{\hbar}{i} a^j \nabla_j + \hat{m} c a^0 \right) \psi = 0 \quad . \quad . \quad . \quad . \quad . \quad (20)$$

¹⁾ A. M. DIRAC, Wave equations in conformal space. Annals of Math. 37, 429—442 (1936).

O. VELEN, A conformal wave equation. Proc. Nat. Acad. of Sci. 21, 484—487 (1935).

²⁾ L. c. S. 442.

In einer konformen Geometrie muss $'\alpha^j = (-g)^{1/8} \alpha^j$ verwendet werden statt α^j , da

$$' \alpha^{(h} ' \alpha^{i)} = \mathfrak{G}^{hi} (21)$$

ist und nur \mathfrak{G}^{hi} verfügbar ist. Dann lautet die Gleichung aber

$$\left(\frac{h}{i} (-g)^{1/8} \alpha^j \nabla_j + \overset{\circ}{m} (-g)^{1/8} c ' \alpha^0 \right) \psi = 0 (22)$$

mit

$$' \alpha^0 = ' \alpha^{[1} ' \alpha^2 ' \alpha^3 ' \alpha^4] = \alpha^0 (23)$$

und diese Gleichung ist bei konstanter Masse im zweiten Term nicht mehr konforminvariant, da $' \alpha^0$ das Gewicht Null hat. Nach unserem Ansatz ist nun aber gerade $\overset{\circ}{m} = \hat{m} (-g)^{1/8}$ konforminvariant, und wir gelangen also sofort zur konforminvarianten DIRACschen Gleichung

$$\left(\frac{h}{i} ' \alpha^j \nabla_j + \overset{\circ}{m} c ' \alpha^0 \right) \psi = 0 (24)$$

Die Gleichung ist bis auf den Einfluss eines eventuellen Vektors P_i in ∇_j identisch mit der gewöhnlichen DIRACschen Gleichung, sodass man sagen kann, dass die gewöhnliche DIRACsche Gleichung, die bekannte Ersetzung von ∂_j durch ∇_j beiseite gelassen, schon konforminvariant ist, sofern man Konforminvarianz in unserem Sinne auffasst und die richtige Massetransformation einführt.

Bei Verwendung der konformen Gleichung muss man natürlich erwarten, dass $\bar{\psi} \psi$ die konforminvariante elektrische Wahrscheinlichkeitsdichte ϱ vom Gewicht $\frac{3}{4}$ darstellt, d. h. ψ muss als Dichte vom Gewicht $\frac{3}{8}$ normiert werden. Da das dreidimensionale Raumelement $d\omega_3$ eine Dichte vom Gewicht $-\frac{3}{4}$ ist, hat $\varrho d\omega_3$ richtig das Gewicht Null. Dann hat aber auch tatsächlich $\bar{\psi} ' \alpha^h \psi$ das Gewicht $+1$, wie es von dem Stromvektor \mathfrak{S}^h verlangt wird.

Bio-physics. — *Notes on the structure of the wall of algae of the genus Halicystis.* By G. VAN ITERSON Jr.

(Communicated at the meeting of October 31, 1936).

In his treatise "Ueber den Bau und die Fortpflanzung von *Halicystis* (Areschoug) und *Valonia* (Ginnani)" (*Botanische Zeitung* 65 I, 1907, S. 137—185) P. KUCKUCK described the wall of *Halicystis ovalis*, which he had gathered at the coast of Helgoland. He observed the following about it:

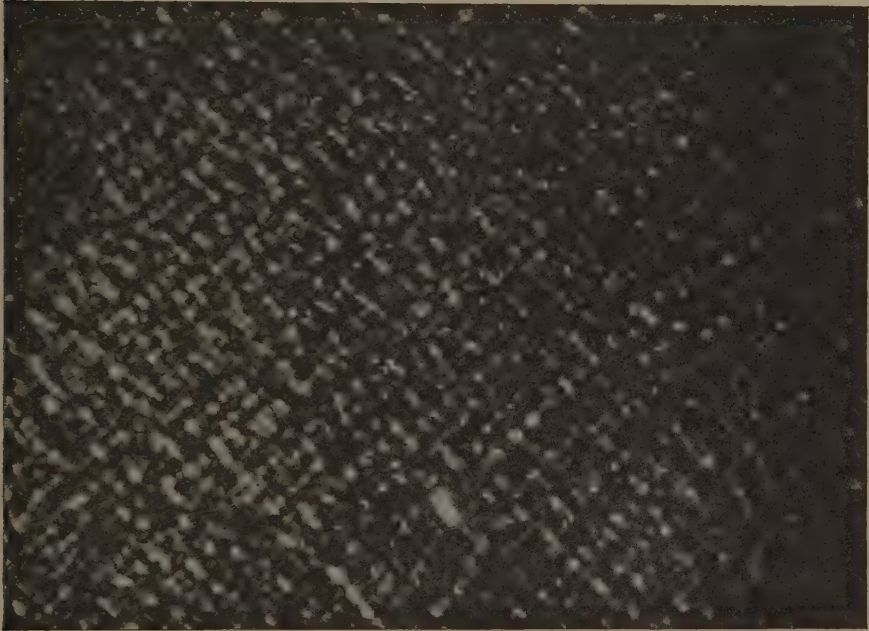
"Die Membran ist 10—12 μ dick und ohne sichtbare Struktur, wie schon Agardh angibt. Auch bei Behandlung mit verschiedenen Reagenzien und Farbstoffen bleibt sie strukturlos, nur werden beim Quellen in Kalilauge im Profil einige wenige Schichtungsstreifen sichtbar, die zeigen, dass die Membran von älteren Blasen aus etwa 2—3 Lagen bestehen kann, die durch eine zarte, weniger dichte Schicht getrennt sind. Lyngbyes Angabe, dass die Membran "sub lente minutissime punctata" sei, ist irrtümlich wie schon Areschoug zeigt. Die Membran gibt die bekannten Zellulosereaktionen".

When I attempted a few years ago to obtain some *Halicystis ovalis* from Helgoland, to study the wall more closely, I was informed that these one-celled Chlorophyceae had been found there regularly for many years, but that lately they were no longer to be found. During a visit to the Hopkins Marine Station at Pacific Grove, California, in the summer of 1933, I was surprised to find there, in an aquarium, a species of *Halicystis* which was collected by Mr. HOLLENBERG near Monterey (California) and was used for physiological investigations. Presumably this species was identical to *H. ovalis*, but it might have been *Halicystis Osterhoutii*, which was reported from several stations near Monterey (vide: L. R. BLINCKS and A. H. BLINCKS, Two genera of algae new to Bermuda, *Bull. Torrey Bot. Club* 57, 389—396, 1931). A very cursory investigation with the polarization microscope showed me that the wall has more interesting features than KUCKUCK had observed.

Recently I received from Dr. L. R. BLINCKS a few dried cells of *Halicystis Osterhoutii* which he had gathered in Bermuda. The alga is found there, namely, washed up on some of the coasts, as beautiful transparent cells up to two or three centimeters in diameter, and known as "sea bottles". The Bermudan species reaches a larger size than *H. ovalis*. The cells received from Dr. BLINCKS indeed were much larger than those I studied in Hopkins Marine Station. With the walls of the dry cells after they were boiled I

found the same optical phenomena as with the smaller bladders from the Pacific. Although my studies are far from complete (I lacked the necessary material), I think it is well to publish a few of my observations.

On viewing the cell walls of *Halicystis* between crossed nicols, I found that the wall shows a very striking aspect. We refer here to the included micro-photo prepared by my assistant, Miss Dr. ALIDA C. SLOEP. One will observe the lightening of the field of vision in spots. These spots are in



reality short light-giving band sections which alternate with dark sections. It is remarkable that many band sections are placed in line, which produces intermittent stripes. These stripes occur in two directions, perpendicular to each other. The directions halve the angles between the vibrational directions of the polarizer and the analyzer, viz. in a normal position of the nicols form angles of 45° and 135° with a vertical. Especially at the points of intersection of two lighting bands the light is strong. I was able to observe further, by alternately focusing exactly on various heights in the wall, that the two systems of band sections are not situated in the same plane, but in two different planes very close above each other.

When the preparation is rotated over 360° , the two stripe systems are most clearly visible in four mutually perpendicular positions, and disappear, almost but not completely, in four positions in between. In the photo they are shown in the most clear position. During the rotation the stripes do not change their direction, but only their intensity. Inserting a $\frac{1}{4} \lambda$ -plate when the stripe systems are in the most clear position, or a gypsum plate red 1st order, produces an image that is still more clear. With gypsum

red 1st order, in one direction blue band sections appear and in the other, yellow; both band sections are alternated by red stripes.

The image, however, is not regular at all, and it changes also, of course, on rotating the object. Moreover, the whole field of vision becomes coloured slightly blue in two positions, and in the two positions perpendicular to the former, slightly yellow.

The images obtained by exactly focusing on a single layer with band sections reminded me strongly of the images which I obtained some time ago when I smeared a solution of the cadmium salt of glyceric phosphoric acid on a slide, and observed the result some time afterwards between crossed nicols, with a gypsum plate red 1st order inserted (vide "Some remarkable properties of a double refracting liquid", Proc. Royal Acad. Amsterdam, 37, 367—376, 1934). I assume from this that the double-refracting matter is present as a series of crystallites placed in waves, and that there are two layers, with crystallites so placed, present closely above each other, while the directions of the waves in these two layers are perpendicular to each other. This supposition, however, is not more than a working hypothesis.

Also at the folds of folded walls of *Halicystis* a structure is visible, when viewed between crossed nicols, and I have used such folds in the beginning to enlighten myself further about this structure. In this manner I have especially investigated a few young walls. To investigate older walls, however, I have made use of cross-cuts, since I suspected that on the folds of thicker walls double refraction would occur which must be ascribed to tensions. I prepared these cross-cuts of cell walls which I had mounted in paraffin. The slices of paraffin obtained by cutting with the microtome were not pasted to the slide, but I dissolved the paraffin in chloroform, filtered the chloroform in which the cuts were floating through a black micro-filter paper (diam. approx. 8 mm), onto which I collected the cuts. I used for this a special micro-filtration apparatus which need not be described here; suffice it to state that the small filter paper is placed as a flat disk on the apparatus and is not folded. The cuts could be loosened easily by moving the reversed filter paper, with a pair of pincers, in a drop of water on the slide. In such a way I could bring tens of cuts under one cover glass, they could be viewed under the microscope, and be subjected to the action of chemicals.

It now appeared that at the outer side of the walls a very thin cuticula is present (naturally it is not proved that this cuticula can be considered as identical to the cuticula of higher plants), and that under this thin cuticula a strongly double-refracting thin layer is present which is followed by a much thicker, practically non-double-refracting layer in older walls. The longest axis of the index ellipse of refraction of the double-refracting layer is placed parallel to the surface of the wall.

The outer surface of the wall is not flat, but slightly waved; sporadically, distinct conical protuberances appear. It is remarkable that the double-

refracting matter seems usually to be more concentrated along the troughs of the waves than near the crests. Sometimes it may be observed that this matter penetrates also deeper into the wall, often as triangular figures of which the bases are placed along the cuticula and the tops directed toward the interior, but thin walls may be also spotted irregularly, which hints at another distribution of the double-refracting matter. Always, however, the most intense double refraction is observed immediately under the cuticula. I estimate the continuous layer of this matter at approximately twice the thickness of the cuticula; the latter I estimate at approximately 1.5μ . The non- or weakly double-refracting layer, which follows on that of the double-refracting matter in the direction of the interior, may become, in the case of large cells, as thick as 30μ .

If swelling agents are applied on cross-cuts, it appears that the layer containing the double-refracting matter swells very easily; the wider layer under this swells less, yet increases to two or three times its thickness. This has been found for the swelling in 50 % alkali, cupric oxide ammonia, 55 % sulphuric acid and chlorozinc solution; the swelling in chloral hydrate is somewhat more difficult. Chloro-zinc iodine solution, especially, gives clear images.

On swelling, the original double refraction disappears practically instantaneously; the systems of bands soon cannot be seen any more in a top view, and in the cross-cuts the double-refracting sub-cuticular layer too almost immediately disappears. It is notable that the inner part of the wall may then become strongly double-refracting; the longest axis of the index ellipse of refraction is perpendicular to the surface of the wall. I consider it probable that this double refraction is tensional, due to tensions produced by the swelling.

It is of importance that a lamellae-structure becomes visible in the last mentioned part of the swollen cross-cuts; with older cell walls I counted from 50 to 60 lamellae, of which some were apparent than others. Occasionally I received the impression that a structure of coarser layers must be accepted as present (in KUCKUCK's sense), but often this was not to be observed, and I do not consider this structure as real.

On swelling, the cuticula is lifted up and in the part of the swollen wall situated between the cuticula and the wall layer containing the lamellae there is no structure visible. Especially with the swelling in chloral hydrate it may be observed how in the beginning the cuticula removes itself from the wall in lens-shaped bubbles; the blisters formed in this way are filled with non-double-refracting matter. The outer wall lamellae of the thick layer do not remain unbroken in the case of the heavily swollen walls, but are split by wedgeshaped cuts across the lamellae penetrating through ten or more of these, towards the interior. I consider these cracks as artificial ones (supported by the fact that lamellae which are most conspicuous continue on the other side of the cracks), which may be compared with the transverse cracks I described some time ago (Biologische Inleiding tot het Cellulose-symposium,

Chem. Weekblad 30, 2—19, 1933) in the case of fibres of which cross-cuts have been prepared along chemical lines, according to the method of J. WIESNER (Unt. üb. die Organisation der veget. Zellhaut, Sitz. ber. Akad. Wiss. Wien, Abt. I, 93, 17—81, 1886) and M. A. EL KELANEY and G. O. SEARLE (The Chemical Sectioning of Plant Fibres, Proc. Roy. Soc. London 106, 357—363, 1930). The fact that the cracks in the swollen wall of *Halicystis* occur only on the outer side indicates, to my mind, that the outer lamellae have been stretched by the growth of the wall to a larger extent than is the case with the inner lamellae, since in swelling agents a great stretch is followed by a great shrinkage (as I shall demonstrate elsewhere).

As far as the chemical nature of the wall substances is concerned, the following may be communicated.

The wall in a top view shows a strong blue colouration with a not too diluted solution of iodo-potassium iodide (with a diluted solution it becomes yellow). The wall, therefore, contains a compound which may be called *amyloid*, but I am not convinced that the "amyloid" of *Halicystis* is identical to the amyloid of higher plants. On the contrary, I consider it very probable that we have to deal here with another matter. It is interesting that H. ZIEGENSPECK, who made a special study of the occurrence of amyloid (see especially: Ueber Zwischenprodukte des Aufbaues von Kohlenhydrat-Zellwänden und deren mechanische Eigenschaften, Bot. Arch. 9, 297—376, 1925), mentions three cases of the occurrence of this substance in the cell walls of algae, viz. in *Microspora*, in *Conferva*, and in the ring of *Oedogonium* (the latter instance ZIEGENSPECK derived from NÄGELI and SCHWENDENER: „Das Mikroskop"). As far as is known to me, there occurs no amyloid in the walls of *Siphonales*, and I have ascertained once more that no amyloid can be found in the wall of *Valonia*, of *Chaetomorpha* and of *Cladophora* (these were the only *Siphonocladales*¹) which were at my disposal besides *Halicystis*).

I have also treated cross-cuts of *Halicystis* walls with iodo-potassium iodide, and was able to demonstrate that the thick inner wall built up of lamellae gives a strong amyloid reaction. The cuticula does not give this, naturally, but I got the impression that the double-refracting matter present under this cuticula too does not show the reaction in the beginning. However, as soon as the concentration of the iodine becomes somewhat high, there also, an intensive colouration appears.

It will be clear that also a solution of chloro-zinc-iodide causes a blue colouration, but the action of this solution produces a marked swelling very quickly, especially of the wall matter under the cuticula, which after the swelling is no longer coloured blue. In every case it not allowed to conclude from this reaction at the presence of cellulose, as probably has been done by KUCKUCK.

¹) I am following here the nomenclature of H. PRINTZ in "Die natürlichen Pflanzenfamilien" 2. Aufl., 3. Bd., Leipzig (1927).

With ruthenium red only a faint red colouration may be observed. Reaction on cross-cuts shows that this colouration is restricted to the cuticula: I could not make out whether the colouring substance (which must be considered, presumably, as a pectinous matter or as another cell wall matter derived from a poly-uronic acid) is present in or immediately under the cuticula. With certainty I could determine that the material which shows the strong double refraction does not give the reaction with ruthenium red, for this material lies slightly deeper in the walls, toward the interior, than the constituent which may be coloured with ruthenium red.

With coralin soda solution, and also with a solution of reso-blue (acc. to TSWETT) and with the acetic acid solution of marine blue (acc. to L. MANGIN) I could observe that the wall gives a strong so-called *callose* reaction. I wish to state, however, that the conception "*callose*" has been very insufficiently defined, and I doubt seriously whether all cell wall substance which shows these reactions is chemically the same matter. I even consider it possible that the same material which above has been indicated as amyloid will give also the callose reactions. In the cross-cuts I was not able to observe any difference in intensity of the callose reactions between the various wall layers.

Further, I let a mixture of dyes react, which mixture is used for "*metachromatic*" colourations, namely a solution consisting of the following: benzobrown 0.5 g, oxamineblue 0.5 g, sodium carbonate 0.5 g, in 100 cm³ of water. A distinct difference in colouration was to be seen here: the layer with the many lamellae became blue, the cuticula and also the strongly refracting layer underneath, turned brown.

Finally I treated the wall for three days with a strong solution of cupric oxide ammonia, after that with diluted acetic acid, and finally washed it with water. As was to be expected from the above, no double refraction was visible when the specimen was now viewed from the side; the band system under the cuticula, observed between crossed nicols, seemed to have disappeared. On the folds of wall parts folded double, double refraction could certainly still be observed; I ascribe these to tensions in the folds. The cuticula was visible as a separate layer; the pectin reaction was as weak as that on the untreated wall. It appeared further that the callose reactions, as well as the amyloid reactions, gave positive results with the wall treated with cupric oxide ammonia. The first mentioned reactions showed nothing of importance. On the reaction of iodine solution (I received the impression that much iodine was necessary to make it start) a remarkable structure became visible; the blue colouring appeared to be concentrated in nodal points from which small blue bands radiated, which connected the nodes to each other. I wish to call attention to the fact, however, that in the foregoing it was stated that cross cracks occurred in the wall, perpendicular to the wall surface, after intense swelling. On viewing swollen walls from the side, one will therefore look through layers of different thickness. Presumably the just ment-

ioned distribution of the amyloid has something to do with this, but I dare not decide whether there are no other reasons why this structure became visible.

From the foregoing it follows that the thick inner wall layer of the larger cells of *Halicystis*, which are built up of lamellae, does not consist of cellulose, but presumably of a mixture of amyloid and callose, or of a cell wall substance which shows both the reactions of amyloid and callose. Besides the lamellae structure nothing particular was shown by this thick layer. The lamellae are presumably visible because of a difference in content of water on the outer and the inner side of a lamella, but the layer is further probably completely homogeneous and optically isotropical when not under tension.

The nature of the strongly double-refracting substance which is present as a thin layer under the cuticula could not be ascertained; we shall return to this further on.

In any case, it becomes clear now why the mechanical nature of the wall of *Halicystis* differs completely from the one of *Valonia*. In a soaked condition the wall of *Halicystis* is slightly elastical and extremely supple; the wall of *Valonia* is coarse, very slightly elastic and paper-like.

The mechanical properties of the wall of *Halicystis* may be regarded as a consequence of the uniform condition of the thick inner wall layer. We call to mind here that the wall of *Valonia* consists of numerous lamellae built up of cellulose fibrillae which course, in two successive lamellae, along directions that cross each other, which structure must cause a great stiffness (see, among others, what I communicated about the wall of this one-celled alga in my lecture: "Introduction to the cellulose symposium", Chem. Weekbl. 30, 2—19, 1933).

Finally I wish to state that on diaphragming strongly and focusing sharply on the various wall layers of *Halicystis*, also in regular light, the impression is given that the wall has still another structure. This structure is best designated as "fine grained", and I can subscribe LYNGBYE's statement, notwithstanding KUCKUCK's denouncement.

If I connect the previous results with what is known of related algae, the following may be stated.

In the first place the poverty in pectin is striking, compared, for instance, with the richness of the walls of *Siphonales* in this cell wall matter. I refer here to R. MIRANDE, "Recherches sur la composition chimique de la membrane et le morcellement du thalle chez les Siphonales" (Ann. Sc. nat., botanique 9e sér., 18, 147—264, 1913). I should like to observe, however, that I consider it very probable that the "composés pectiques" of MIRANDE were partly compounds of poly-uronic acids, which we, at this time, should not call pectinous material any more. An investigation of the cell wall material of *Siphonales* in the light of recent observations in this field (I refer here to the thesis of my pupil H. A. FRANKEN: The presence, preparation,

and properties of uronic acids, and some related acids found in nature. Delft, 1934) promises really important results.

The poverty in pectinous material in the wall of *Halicystis* is especially interesting since *Valonia* too is poor in this material. With *Valonia* I found the material in, or directly under, the cuticula, thus in the same place as in *Halicystis*, but *Valonia* is somewhat richer in this cell wall material.

It is noteworthy, however, that *Valonia* is rich in cellulose, and that the presence of this in *Halicystis* is doubtful. It is certain that cellulose fibrillae which form the principle structural constituent of the walls of *Valonia* (see especially C. CORRENS: Zur Kenntn. der inneren Struktur einiger Algenmembranen, ZIMMERMANN's Beitr. Morph. u. Physiol. Pflanzen. 1, 260—305, 1893, and F. BRAND, Ueb. die Faserstruktur der Cladophora-Membran, Ber. d. D. bot. Ges. 24, 64—71, 1906), are lacking in *Halicystis*. The thick inner wall, rich in lamellae, which for the larger cells of *Halicystis* *Osterhoutii* forms 90 % of the cell wall, certainly does not consist — as I have said above — of cellulose. Yet I consider it not excluded that the double-refracting material, which we found under the cuticula, consists of cellulose crystallites which are imbedded in such a way in the material capable of swelling that with the swelling a disarrangement of the crystallites occurs, owing to which the double refraction disappears. This hypothesis is especially tempting, since with it a certain analogy with the cell wall of *Valonia* can be indicated. With *Halicystis* as well as with *Valonia* there should lie, inside the cuticula, an extremely thin layer which is rich in pectins, thereupon there should follow two lamellae with cellulose crystallites, which in the two lamellae should be arranged in directions which are practically perpendicular to each other. I note here that I sometimes found images, in viewing the wall of *Valonia* between crossed nicols, which reminded me of the photo shown above, which images I ascribe to the fact that in *Valonia* (also in the direction parallel to the wall) waves may be present in the fibrillae. The difference in the wall of the species of both genera should then be found herein: with *Halicystis* no new identical lamellae follow the two cellulose-containing lamellae, but numerous lamellae of amyloid should be found deposited on them from the inside; with *Valonia*, on the contrary, many wall lamellae with cellulose crystallites should be found next to the first two lamellae. One could call attention here to the fact that amyloid is considered by ZIEGENSPECK as an in-between product in the building up of cellulose, and it could be accepted, therefore, that in the lamellae of *Valonia*, which are situated more towards the interior, the synthetic process of cellulose formation goes one step further than is the case in the corresponding lamellae of *Halicystis*.

However attractive this explanation may be, I must state that I have not succeeded in settling for certain whether the double-refracting matter of *Halicystis* really consists of cellulose. It makes up only a small percentage of the wall, which is rich in amyloid, and which makes reacting for cellulose impossible. In this connection we do not refrain from warning the reader

that our photo may cause a flattering impression of the significance of the double-refracting matter; this significance can be seen only in the cross-cuts.

It is my conviction that a closer study of the wall of *Halicystis* and of its development may lead still to important points of view. For instance, I have already determined that the layer built up of lamellae of amyloid with young specimens is developed to a much smaller extent than with older ones, and above I called attention to it that the older lamellae are in all probability strongly stretched when the cells become larger; the deposition of the lamellae occurs therefore very probably by *apposition*. It is further noteworthy that the "stripe structure" appeared to be much finer with young walls than with older ones. This probably means that the waves of the series of crystallites become longer and higher as the wall grows older, from which, however, must be concluded that a uniform growth of these waves takes place by *intussusception*.

Finally I wish to state that also a chemical study of the wall of *Halicystis* will be found very promising, since in that wall a material is found which gives strong amyloid and callose reactions, while cellulose and pectinous matter are practically absent.

In any case it will be clear that the cell wall of *Halicystis*, which algae have been the subject of such interesting physiological investigations during the last years (among others, those of W. J. V. OSTERHOUT and M. J. DORCAS and of L. R. BLINCKS), is worth a closer study.

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Delft, July 1936.

Plantkunde. — *De grenzen der bloeibaarheid en het groeien van den Iris-bol.* IIB. (with summary). Door A. H. BLAAUW, IDA LUYTEN en ANNIE M. HARTSEMA. (Mededeeling N^o. 50 van het Laboratorium voor Plantenphysiologisch Onderzoek te Wageningen.)

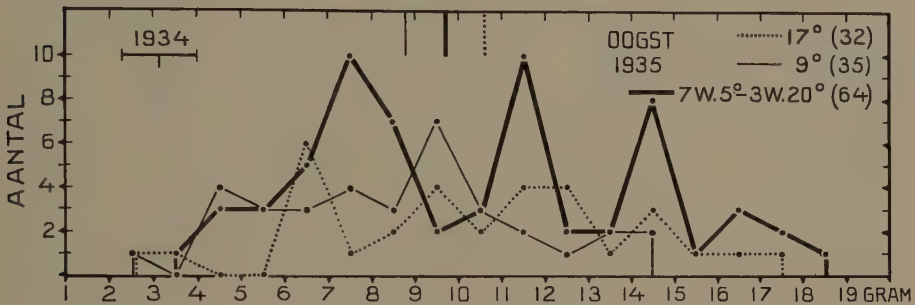
(Communicated at the meeting of October 31, 1936).

In het voorafgaande nummer der Proceedings werden (onder IIA) de bloei-resultaten na de proeven van 1934—'35 en 1935—'36 gegeven. Moeilijker is het bij een vermijden van den bloemaanleg tevens een behoorlijken groei te bewerken. De uitkomsten van dien groei worden hier nader besproken onder verwijzing naar de cijfers, die reeds in de 5 tabellen van IIA zijn opgenomen.

Groei.

In de cultuur worden de bollen gezeefd en aldus gesorteerd naar den omtrek der bollen. Dit sorteeran van bollen en het uitdrukken van de

grootte volgens de zeef-gaten waar de bollen wel en waar ze niet doorheen vallen, is natuurlijk een vrij grove methode; maar het gaat ook om het



Variatie in 't gewicht bij den oogst van ronde bollen (Imperator, 1935) na 3 verschillende behandelingen in 1934. Links boven: gewichtsvariatie, waarbinnen alle bollen lagen bij het begin van de proef, met het gemiddelde op 3,12 g. In het midden, boven de 3 curven met wijd geworden variatie, het gemiddelde gewicht van deze 3 groepen in 1935. Geheel rechts de behandeling (zomer '34) en het aantal beschikbare ronde bollen in '35.

sorteeren van enorme hoeveelheden. Bij laboratorium-proeven wegen wij de bollen stuk voor stuk; kiezen als uitgangspunt vrij nauwe grenzen waarbinnen alle proefbollen moeten liggen en wegen het volgende jaar den oogst weer stuk voor stuk. Het gemiddelde gewicht per ronden bol vindt men in de 4e kolom der tabellen. In tab. 1, 2 en 3 is in kol. 5 daarbij vermeld \pm de gemidd. omtrek in cms. Deze is ongeveer afgeleid uit een proefondervindelijk samengestelde curve, die het verband tusschen gewicht en omtrek ongeveer weergaf. In tab. 4 en 5 is bij den oogst de omtrek van alle bollen gemeten en de gemidd. omtrek precies aangegeven. Deze omtrek wordt er bijgevoegd omdat de cultuur en de handel gewend zijn de grootte der bollen hierin uit te drukken. Bij dat gemiddelde moet dus bedacht worden, dat de omtrek van 40 à 50 % der bollen daarboven uitgaat. Door het wegen van alle exemplaren konden wij vaststellen, dat na alle soorten behandeling, het gewicht der nieuw-geoogste bollen zeer *aanzienlijk varieert*, terwijl wij uitgingen van een groep binnen *nauwe grenzen*. In bijgaande figuur wordt dit geïllustreerd voor een zoo gunstige behandeling als 17° en voor bollen, die op 2 manieren koude hadden ontvangen. Door de weinige beschikbare ronde bollen vertoonen de curven sterke schommelingen, als wij op de abscis de aantallen afzetten met slechts 1 gram opklimmend. De laatste behandeling geeft hooger toppen, echter alleen doordat hier bijna een dubbel aantal ronde bollen ter beschikking stond. Het belangrijkste voor ons is echter, dat na kou-behandeling de variatie niet erger wordt dan ze evenzeer na 17° is. — Bij het vergelijken van de groei-uitkomsten zullen wij ons nu beperken tot het gemiddelde *gewicht* per ronden bol, omdat dit veel nauwkeuriger en scherper de verschillen in de opbrengst doet naar voren komen. Men lette daarbij op het uitgangs-

gewicht bij het planten. In tab. 1 en 2 is, als men 10 weken gelijke temperatuur geeft, de oogst het best na 17° C.; na lager en hoger temperatuur dalende. In tab. 2 is het gewicht na 13° C. bijzonder laag; ten deele kan dit toeval zijn, daar hier door de vele bloeiërs maar 12 ronde over bleven. Toch hebben wij reeds meer ervaren, dat 13° C. ongunstiger resultaten opleverde, dan men zou verwachten; dit kan ook daaraan liggen, dat deze temperatuur het vroeg opkomen van het blad het meest bevordert en daardoor het loof het meest van koude nachten te lijden heeft. Veiligheidshalve raden wij in ieder geval een behandelen met 13° C. af (zie voorloopig advies in Meded. 42); te meer waar we zien, dat bij *Imperator* na 13° zelfs meer bloeiërs dan na 17° C. kunnen optreden.

De vele soorten van behandeling volgens tab. 2 gaven ook in den groei ('t gewicht) geen sterke differentiatie. Het overgrootste deel der behandelingen levert een flink gewicht op, dat het effect van 17° (en 20°) evenaart en soms dat van 17° nog even overtreft. Het belangrijkste is voor ons doel het gewicht na de behandeling, die het laagste % bloemen oplevert. Na 10 w. 5° met 22 % is het gewicht (6.67 g) bepaald veel te laag; maar 7 w. 5° gevolgd door 3 w. 20° geeft met 30 % bloem een gewicht van 9.24 g, wat nog zeer behoorlijk is. Het blijkt dus, dat 3 w. 20° inplaats van 5°, aan het slot gegeven, het gewicht aanzienlijk verbetert en het bloeipercantage maar weinig verhoogt. Vooral op deze soort temperatuurcombinatie zullen wij later voortbouwen.

Nog wil ik in deze tabel wijzen op het hoogste gewicht na 3 w. 31° gevolgd door 13° C. In gevallen waar voor bloemaanleg nog geen vrees bestaat, dus bij zeer kleine bollen, zou een dergelijke behandeling met een korten tijd 31°, gevolgd door 13° C., of waarschijnlijk zelfs beter 17° C., wellicht met succes voor den groei zijn toe te passen. Maar die zeer hoge temperatuur dure niet te lang (zie bijv. 5 w. 31—5 w. 13°), — zoodat bijv. 2 w. 31° gevolgd door 17° voor de kleinste bollen *wel* zou te probeeren zijn in de cultuur. Blijven wij eerst nog bij *Imperator* in het volgende jaar (tab. 4), dan geeft hier steeds 17° een beter gewicht (12.78 g) dan in het vorige jaar (10.63), terwijl 21 d. 9° en 5° en 11 d. 2° gevolgd door 17° even gunstig zijn. Voor den bloei biedt dit in deze partij allerm minst voordeel boven 17° steeds, maar op zichzelf is 't opvallend, dat de groei er geenerlei nadeel van ondervindt. Totaal achterwege blijft de bloei alleen na 10 w. 5° en 51 d. 5° + 18 d. 17°. In het eerste geval is de groei zeer slecht (5.64 g), maar door slechts 18 d. 17° aan het slot wordt het gewicht reeds vrij wat beter (9.62 g), hoewel nog niet voldoende.

Tab. 3 geeft een vergelijking met de variëteit *Wedgwood*. In de gunstigste gevallen wordt hier een gemiddelde van 25 g bereikt, wat zeer hoog is, maar dit is steeds maar vastgesteld aan de weinige ronde bollen, die na die temperatuur niet gebloeid hadden. In elk geval is ook hier 17° C. de goede bewaartemperatuur, als men alleen op groei let. Zelfs 10 w. 9° en 7° geven met 21.5 en 20.0 g nog een behoorlijken groei en een zeer sterke reductie van den bloei. Volkomen bloei-uitsluiting geeft 10 w. 5°

met een gewicht van 17.5 g en verder 7 w. 5° + 3 w. 20°, wegende 18.89 g.

Gaan we ten slotte op tab. 5 over, dan zien we, dat in 1935—'36 Wedgwood van gelijke zwaarte niet alleen veel minder bloeide, maar ook in groei ver achter staat bij 1934—'35. Daarbij behoort steeds 17° nog tot de besten, waarbij het opvallend is, dat 3 w. 5° of 2° vóór 17° het gewicht van 17° niet benadeelen, maar eer begunstigen, zooals ook tab. 4 vertoonde. Dit jaar schijnt zoo'n korte kou-behandeling reeds gunstig tegen den bloei, maar in 't algemeen is dat volkomen onvoldoende (zie tab. 3). Terwijl in dit jaar de bloei-dispositie dus zwak was, wijzen de vroegere resultaten er op, dat voor een krachtig vermijden van bloeiers in de *verschillende* jaren een *directe* toepassing van 7 weken 5° C. gunstig werkt; dat het hierdoor ontstane nadeel voor den groei reeds door 3 weken nabehandeling met 17° (of 20°) merkbaar getemperd wordt; maar dat getracht moet worden den groei verder op te voeren *na* de kou-behandeling, die vereischt is om de dispositie voor bloemaanleg te onderdrukken.

Wageningen, September 1936.

SUMMARY.

The limit of flowerformation and the growth of Iris-bulbs. II.

The results of the flowering-percentage and the growth (increase in weight) of round Iris bulbs in 1934—'35 and 1935—'36 are summarized as a continuation of the Communications 41 and 42. We have to deal here with bulbs of such a size that it is uncertain what percentage will flower and where it is necessary to avoid this flowering as much as possible. This is done in order that the round bulbs may further increase in size, so that a following year they will be sufficiently large to yield practically 100 % of flowers. In the tables 1—5 should be particularly noticed the treatment (column 1), the percentage of flowers (column 3) and the average weight of the gathered round bulbs, which consequently have not flowered (column 4).

1. At equal size and treatment one year yields far more flowers in the early part of the following summer than another (Compare tables 2 and 3 to 4 and 5). Already the conditions in the ground before lifting have a large influence on the disposition either to form flower or not, though flower-formation will not take place in the ground until Jan.—March (in case of *Imperator*).

2. Between lifting and planting the disposition to form flower may be increased (by 23° and higher) or strongly reduced (by low temperatures). For cultivation the latter treatment is of importance in case of doubtful sizes, but there is the difficulty that the bulb also has to grow sufficiently. The optimum for this is about 17°, but for sufficient avoiding flower-

formation in most years and for the larger doubtful sizes a much lower temperature is needed. Since in some years already during lifting the bulbs unexpectedly possess a far greater disposition to flower, a fairly powerful cold-treatment has to be applied in order to avoid surprises. Now it is a question whether we shall yet succeed in obtaining a sufficient growth of the bulb.

3. It has become apparent that in the period between lifting and planting the bulbs in the first weeks after lifting are more sensitive to any influence on their disposition to form flower than in the last weeks. See particularly the tables 2 and 3, where 7 weeks 5° preceded by 3 weeks 20° still gives many flowers, but applied at once and followed by 3 weeks 20° yields far less flower. Moreover, Wedgwood is much more sensitive to this treatment than Imperator, though the flowering-ability yet was very strong with that group (table 3).

4. For Imperator (tables 1, 2 and 4) the flowering-percentage was after 10 weeks 17° (and 20°) often lower than after 10 weeks 13° . Also when during a short period, e.g. 3 weeks, a low temperature is given, followed by 17° , often more flower occurs than after 17° only. To this phenomenon we shall refer again another year. For Wedgwood it has not been observed so far. It is a warning that for flower-reduction 13° C. — or 3 weeks cold followed by 17° C., at any rate for Imperator and kindred bulbs, offers no advantage over 17° C. In order to avoid sufficiently the flower-formation in the various years it is necessary to apply the cold *directly* after lifting and to continue it for some weeks. In the experiments described here particularly 7 weeks 5° was effectual.

5. As far as the growth of the bulbs is concerned, about 17° C. is the most suitable temperature when the bulbs are still very small. With larger bulbs the percentage of flowers becomes too high. In order to prevent this, a low temperature has to precede. However, this makes the increase in weight smaller as well. It appeared from these experiments that if, instead of 10 weeks cold (5°), only 7 weeks 5° is applied, followed by 3 weeks 20° or rather 17° C. (it actually was only 18 days), the increase in weight is already greatly improved and the flowering-percentage is hardly raised (table 2) or not at all (table 3). That the increase in weight after 17° is larger than after a low temperature is due to the fact that the foliage-leaves, as to number and length, grow better in spring, and consequently the assimilation is directly quantitatively influenced.

On this basis — some weeks cold for flower-reduction, followed by some weeks 17° in order to improve the growth — these experiments are now being continued, in hope that thus a method may be obtained for practical application.

Medicine. — *Age and rate of decrease of red blood cells before and after liver-treatment of pernicious anemia*¹⁾. By L. S. ORNSTEIN and J. F. SCHOUTEN. (Communication from the Physical Institute of the University of Utrecht.)

(Communicated at the meeting of October 31, 1936).

§ 1. *Clinical Phenomena.*

The concentration of bilirubin in the blood serum of patients suffering from pernicious anemia is influenced by liver-treatment in a marked and characteristic way. The concentration, generally being higher than normal before the treatment, diminishes rapidly under influence of the therapy (MINOT and MURPHY²⁾. According to KAMERLING³⁾, who measured 8 cases (see Table I), the concentration drops to practically zero

TABLE I.
Concentration of bilirubin in the blood serum of patients with pernicious anemia during intramuscular liver-treatment after KAMERLING.

Weeks after beginning of treatment	0	1	2	3	4	5	6	7	8	9	10	11	12	13
No. 1	5 E	4.4	0.4	—	trace	0	—	—	—	—	—	—	—	—
2	2.3	1.3	0.4	trace	trace	0	0	—	—	—	—	—	—	—
3	2.1	1.2	—	0.5	0	0	0	—	—	—	—	—	—	—
4	3	1.7	0.9	trace	trace	0.3	0.8	0.9	—	—	—	—	—	—
5	1.5	—	—	—	0	0	—	0	0	—	trace	—	—	—
6	1.4	1.1	—	0.4	0.3	0.3	0.3	0.3	—	0	—	0	—	0.7
7	2.5	1.4	1.2	—	0.2	0.1	—	—	0.1	—	—	—	—	—
8	3	—	1.2	1	1	trace	0.3	1.6	2	—	—	—	—	—

within a few weeks and rises again after a time varying between 5 and 13 weeks (see figure 1 and 2).

¹⁾ One of us came to the idea of the possibility of the mathematical treatment of this problem at a lecture delivered by Prof. HYMANS V. D. BERGH at a meeting of the Academy.

²⁾ MINOT and MURPHY, Journ. Am. Med. Ass. Vol. 87, 470 (1926).

³⁾ A. W. C. G. KAMERLING, Ac. Thesis, Utrecht (1936).

Qualitatively this behaviour may be interpreted as follows. The bilirubin in the serum is finally produced when the hemoglobin is destroyed. Its concentration therefore is related to the rate of decrease of the red blood cells. The effect of the treatment is that, at the moment the liver cure commences, the production of "anomalous" cells stops and of "normal" cells starts.

If we assume that the normal cells live appreciably longer than the anomalous cells, one may expect that from the commencement of the cure the concentration of bilirubin will drop quickly. The anomalous cells namely still disappear with the same rate of decrease as before and thus give their contribution to the concentration of bilirubin (Process I). The normal cells, however, meanwhile produced do not die until much later and thus the concentration of bilirubin will not rise until a disappearance of the new cells begins (Process II).

The age of blood cells playing an essential rôle in these processes, it is to be expected that by carefully studying this phenomenon an insight may be obtained in the laws governing the mortality of red blood cells in pathological as well as normal cases.

In the present paper we shall try to draw the mathematical consequences of the qualitative description mentioned above, so as to establish a relation between the concentration of bilirubin in the blood serum and the quantities characteristic of the disappearance of red blood cells. It is possible in this way to obtain a quantitative description of this phenomenon.

Two possible methods present themselves. The first is to calculate the concentration of bilirubin from an assumed form of the function describing the rate of decrease. The second is to try and establish independent of the special form of this function a relation between this function and the resulting concentration of bilirubin.

The second method, proving to be a fruitful one, will be followed. We shall develop it mathematically in § 2, express its meaning in § 3 and apply it to the experimental data in § 4.

§ 2. *Mathematical Formulation.*

a. General.

At a certain moment t the number of red blood cells per unit of volume having an age between T and $T + \Delta T$ is defined by

$$E_{Tt} \Delta T.$$

The total number of cells per unit of volume at the time t is then given by

$$E_t = \int_0^{\infty} E_{Tt} dT (1)$$

If $F_T \Delta t$ represents the probability per cell of age T to die within the interval T and $T + \Delta t$, the general differential-equation becomes

$$\frac{\partial E_{Tt}}{\partial T} + \frac{\partial E_{Tt}}{\partial t} + F_T E_{Tt} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This equation is derived by considering that the number $E_{T+dt, t+dt}$ results from the number $E_{T,t}$ and is equal to those surviving. Thus

$$E_{T+dt, t+dt} = E_{T,t} (1 - F_T dt).$$

The general solution of equation (2) is given by

$$E_{Tt} = f(t-T) e^{-G_T} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

when G_T is defined by $G_T = \int_0^T F_\xi d\xi$.

The function $f(t-T)$ equals the number of cells formed at the time $t-T$ so that

$$E_{Tt} = E_{0, t-T} e^{-G_T} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We shall now derive the distribution of the cells over the different ages T when this distribution is stationary. The condition is

$$\frac{\partial E_{Tt}}{\partial t} = 0$$

¹⁾ Another and more direct way of obtaining (4) is to regard a certain group of cells of the same age as a function of this age T . Let n_0 represent those formed at time $T=0$, then the number n_T left at age T is

$$n_T = n_0 e^{-G_T} \quad . \quad . \quad . \quad . \quad . \quad (4^*)$$

this being the solution of

$$\frac{dn_T}{dT} = -F_{(T)} n_T.$$

The number of cells of age T at the time t is then given by

$$(n_T)_t = (n_0)_{t-T} e^{-G_T} \quad . \quad . \quad . \quad . \quad . \quad (4^{**})$$

and the total number at the time t

$$(n)_t = \int_0^\infty (n_T)_t dT = \int_0^\infty (n_0)_{t-T} e^{-G_T} dT \quad . \quad . \quad . \quad . \quad (1^*)$$

The “ n -notation” is specially useful for studying a certain group of cells of the same age T as a function of T .

Putting this into equation (2) and using E_{T_s} for the stationary distribution function we get

$$E_{T_s} = E_{0_s} e^{-G_T} \dots \dots \dots (5)$$

in which the integration constant E_{0_s} has the meaning of the birth-rate in the stationary case.

Once this stationary distribution function E_{T_s} is known, the other characteristic quantities may be easily defined and derived.

The *mortality* F_T , is given by

$$F_T = -\frac{\frac{d E_{T_s}}{d T}}{E_{T_s}} \dots \dots \dots (6)$$

The second is W_T , the *probability of age* viz. the apriori probability of dying at the age T . W_T is the product of the probability to attain this age and the mortality at that age, so that

$$W_T = \frac{E_{T_s}}{E_{0_s}} F_T \dots \dots \dots (7)$$

or

$$W_T = -\frac{1}{E_{0_s}} \frac{d E_{T_s}}{d T} \dots \dots \dots (8)$$

The third: the *mean age* τ is defined by the integral

$$\tau = \int_0^{\infty} T W_T dT \dots \dots \dots (9)$$

which after inserting (8) and partially integrating can be transformed into

$$\tau = \frac{1}{E_{0_s}} \int_0^{\infty} E_{T_s} dT \dots \dots \dots (10)$$

and by inserting (5) into

$$\tau = \int_0^{\infty} e^{-G_T} dT \dots \dots \dots (11)$$

If we assume that the concentration of bilirubin at a certain time is directly proportional to the amount of cells dying per unit of volume, we may put for the amount of bilirubin originating from these cells of age T :

$$b_{T_t} = c F_T E_{T_t} \dots \dots \dots (12)$$

where the constant c is determined by the amount of bilirubin produced per red blood cell and by the way in which the bilirubin is removed from the blood.

The total concentration of bilirubin b_t is given by

$$b_t = \int_0^{\infty} b_{Tt} dT = c \int_0^{\infty} F_T E_{Tt} dT. \quad (13)$$

Substituting (4) we get

$$b_t = c \int_0^{\infty} F_T E_{0,t-T} e^{-G_T} dT^1) \quad (14)$$

b. Process I (anomalous cells).

We assume that the anomalous cells are produced at a constant rate till the beginning of the treatment and that no more of such cells are formed after that time ²⁾. Thus ³⁾

$$\left. \begin{aligned} E'_{0,t-T} &= E'_{0,t} & \text{for } T > t \\ E'_{0,t-T} &= 0 & \text{for } T < t \end{aligned} \right\} \quad (15)$$

according to (14)

$$b'_t = c' \int_0^{\infty} F'_T E'_{0,t-T} e^{-G'_T} dT.$$

¹⁾ Equations (13) to (15) hold good only approximately. They can be derived in the following way

$$\frac{db}{dt} = ab + B \int_0^{\infty} F_T E_{Tt} dT \quad (13a)$$

under assumption that the removal of bilirubin is proportional to the concentration. B is the amount of bilirubin produced per red blood cell. In the stationary state $\frac{db}{dt} = 0$ so that

$$b_t = \frac{B}{a} \int_0^{\infty} F_T E_{Tt} dT \quad (13b)$$

which equals (13) if $c = \frac{B}{a}$.

This equation also holds true when $\frac{db}{dt} \ll ab$ which implicates that the mean time in which the bilirubin is removed from the serum is small compared to the mean age of the red blood cells. We assume that this is the case and will therefore use the equation (13).

²⁾ A difficulty arises regarding the choice of the time $t = 0$, because of the time-lag existing between the beginning of the treatment and the beginning of the recovery of the peripheral blood, as judged by the reticulocyte response. This being a matter of some five days, we leave it aside for the present and deliberately choose $t = 0$ at the beginning of the treatment. When more exact measurements should become available, this time-lag ought, however, to be taken into account.

³⁾ From now on all quantities referring to anomalous cells shall be marked by '.

Substituting (15) we obtain

$$b_t = c' E'_{0,s} e^{-G'_t} = b'_0 e^{-G'_t} \quad \dots \quad (16)$$

or, substituting (5)

$$\boxed{E'_{Ts} = \frac{1}{c'} b'_T} \quad \dots \quad (17)$$

The concentration of bilirubin as a function of the time is thus proportional to the statistical partition in age for the stationary state of the red blood cells.

Equations (6), (8) and (10) enable us to express F'_T , W'_T and τ' as a function of b'_T and $\frac{db'_T}{dT}$

$$F'_T = -\frac{\frac{db'_T}{dT}}{b'_T} \quad \dots \quad (18)$$

$$W'_T = -\frac{1}{c' E'_{0,s}} \frac{db'_T}{dT} \quad \dots \quad (19)$$

$$\tau' = \frac{1}{b'_0} \int_0^\infty b'_T dT \quad \dots \quad (20)$$

c. Process II (normal cells).

We assume that the normal cells are produced from the beginning of the treatment at a constant rate. Thus

$$\left. \begin{aligned} E_{0,t-T} &= 0 & \text{for } T > t \\ E_{0,t-T} &= E_{0,s} & \text{for } T < t \end{aligned} \right\} \quad \dots \quad (21)$$

according to (14)

$$b_t = c \int_0^\infty F_T E_{0,t-T} e^{-G_T} dT$$

Substituting (21) we find

$$b_t = b_\infty - c E_{0,s} e^{-G_t} \quad \dots \quad (22)$$

b_∞ representing the ultimate stationary concentration of bilirubin.

Now substitute (6)

$$E_{Ts} = \frac{1}{c} (b_\infty - b_T) \quad \dots \quad (23)$$

Proposition IIb (W_T).

The probability for a normal blood cell to attain an age T and to die then is directly proportional to the derivate of the concentration of bilirubin at the time $t=T$.

Proposition IIc (F_T).

The mortality of the normal blood cell of age T is equal to the negative logarithmic derivate of the function $(b_\infty - b_T)$.

Proposition IId (τ).

The mean age τ of the normal blood cells is equal to the area of the curve $(b_\infty - b_T)$ divided by b_∞ .

§ 4. Application to KAMERLING's experiments.

In Fig. 1 and 2 KAMERLING's cases 4 and 8 are shown together with the functions derived from these. One has to take into account, of course, that these measurements do not claim precision. They intended to give a rough idea of the phenomenon observed. For definite conclusions they should have to be repeated by measurements not taken weekly but daily. The general aspect, however, is so marked that two significant conclusions may none the less be drawn from the experiments. The first is that, as was expected beforehand, the mean age of anomalous cells is much smaller

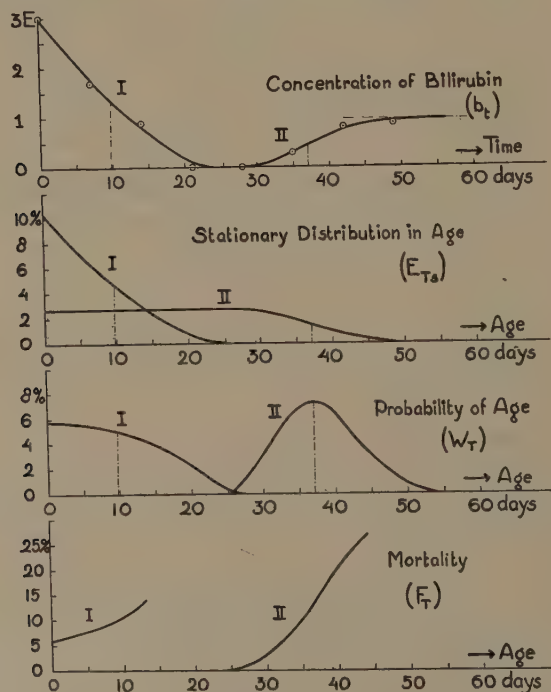


Figure 1 (KAMERLING's case No. 4).

Curves I: red blood cells before treatment (anomalous cells, mean age 10 days).

Curves II: red blood cells after treatment (normal cells, mean age 37 days).

than that of normal cells (see Table II). Whereas anomalous cells have a mean age varying between 9 and 19 days, the normal cells in cases 4 and 8

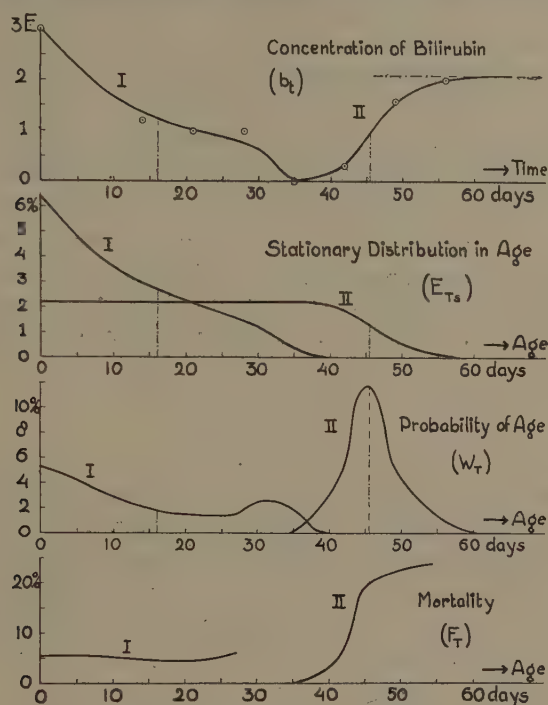


Figure 2 (KAMERLING's case N^o. 8).

Curves I: red blood cells before treatment (anomalous cells, mean age 16 days).

Curves II: red blood cells after treatment (normal cells, mean age 46 days).

TABLE II.

Mean ages of red blood cells before and after liver-treatment.

No. of case	Conc. of bilirubin at $t=0$ (b'_0)	Mean age of cells before treatment (τ')	Mean age of cells after treatment (τ) (calculated value)	Mean age of cells after treatment (τ) (estimated value)	Ultimate concentration of bilirubin (b_∞) (estimated value)
1	5.0 E	11 days	—	greater than 40 days	—
2	2.3	9 "	—	" " 45 "	—
3	2.1	11 "	—	" " 45 "	—
4	3.0	10 "	37 days	—	0.95 E
5	1.5	—	—	" " 70 "	—
6	1.4	19 "	—	" " 90 "	—
7	2.5	14 "	—	" " 60 "	—
8	3.0	16 "	46 days	—	2.1 E

show a mean age of 37 resp. 46 days and in the other cases a mean age which cannot be established from the experiments but which is certainly greater than 40 days. The second is that a marked qualitative difference exists between the rate of mortality of the two kinds of cells. The normal ones have a chance of dying equal to zero during the first stage of their life and do not die until after having reached a certain age, whereas the anomalous ones already have a certain probability to die directly after their origination, this probability remaining practically constant throughout their short life. This leads to the suggestion that the normal cells in the peripheral blood pass through some sort of second maturation process and thus possess an intrinsic age, whereas the anomalous ones remain just the same as they were at the moment of their birth; their death is only ruled by chance and may be readily compared with the disintegration of radioactive atoms.

§ 5. *Summary.*

A simple relation is established between the remarkable change in the concentration of bilirubin in the blood serum of patients with pernicious anemia during liver-treatment and the laws governing the death-rate of red blood cells at each age before as well as after the treatment.

Firstly, the mean age before the treatment varies between 9 and 19 days, whereas after the treatment it is at least 37 days but may in one case very well run up to at least 90 days.

Secondly the rate of mortality is of different type before and after the treatment. The anomalous cells have a mortality practically independent of their age, whereas that of the normal cells is highly dependent upon their age. The anomalous cells thus die at random, the normal cells at an age not differing much from the mean age.

Physics. — *Vapour pressures of liquid krypton.* By J. J. MEIUIZEN and C. A. CROMMELIN. (Abstract of Communication N^o. 245c from the KAMERLINGH ONNES Laboratory.) (Communicated by Prof. W. H. KEESOM.)

(Communicated at the meeting of October 31, 1936).

Vapour pressures of liquid krypton have been measured from the triple point to the critical point.

They can within the limits of experimental error be expressed by the equation:

$$\log p_{\text{norm. atm.}} = \frac{-899.979}{T} - 12.55400 \log T + 31.50311 + 0.0175105 T,$$

in which p represents the vapour pressure at the temperature T .

The fundamental points of krypton are given in the following table:

Fundamental points of krypton.

	$t_{\text{°C}}$	$T_{\text{°K}}$	$p_{\text{norm. atm.}}$
Critical point	— 63.75	209.39	54.182
Boiling point	— 153.21	119.93	1.0000
Triple point	— 157.17	115.97	0.7204

Physics.—*Visual intensity-measurement with the aid of successive contrast.*
By D. VERMEULEN and J. G. HAGEDOORN. (Communication from the Physical Institute of the University of Utrecht.) (Communicated by Prof. L. S. ORNSTEIN.)

(Communicated at the meeting of October 31, 1936).

In visual monochromatic photometry it is necessary to adjust the brightness of a field illuminated by a calibrated lamp to equalise that of a field illuminated by the unknown lightsource. For high accuracy the borderline between the two fields should be quite sharp. In the case of a spectralpyrometer, reflection, deflection and deviation from the law of LAMBERT at the edges of the filament will make it possible to have the image of the filament disappear more or less in the background when the brightnesses are nearly the same. By choosing suitable circumstances¹⁾ these errors can be eliminated to a great extent, but it involves working with very small solid angles. This makes it impossible to observe low brightnesses. To be able to measure at low intensities one would therefore have to have a method where large solid angles may be used.

In this way we arrived at the idea of comparing brightnesses by successive contrast. With the aid of a revolving mirror the images are observed successively through the same optical system. The quality of the image being immaterial, large solid angles may be employed and the image may be concentrated.

A scheme of the method is given in figure 1. With the aid of the

¹⁾ FAIRCHILD and HOOVER, Journ. Opt. Soc. Amer. 7, 543 (1923).

revolving mirror images of the standard lamp and the unknown lightsource are successively formed on the slit of a monochromator.

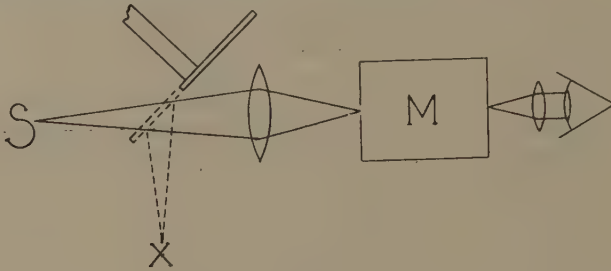


Fig. 1

To test the accuracy and sensitivity of this method an arrangement was made as shown in fig. 2.

On the opening in the screen (9) two images are formed of the same lightsource (1). The brightness of one image can be varied by moving the screen (4), and in this way it is possible to adjust them to equal brightness. We used a green filter instead of a monochromator.

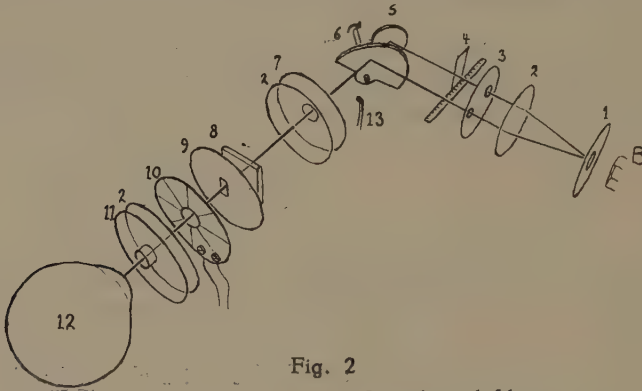


Fig. 2

B. tungsten bandlamp

1. diaphragm with opaline glass

2. lens

3. screen with a large and a small opening

4. movable screen

5. fixed mirror

6. revolving mirror

7. diaphragm

8. coloured filter

9. diaphragm with rectangular opening

10. diaphragm on which glowing filaments of variable brightness are stretched

11. perforated lens

12. the eye of the observer

13. metal strip for audible signal

At very low intensities the image was found with the aid of the filaments on the diaphragm (10). The screen (9) was observed at infinity through the hole and with the aid of the second lens, and at the same time the filaments were seen at infinity with the aid of both lenses.

Each time the images were interchanged in the same sequence a metal strip (13) gave a sharp audible signal. This enabled us to move the screen (4) in the right direction to equalise the flow of energy. It also gave the exact moment of interchangement, when the brightnesses were nearly the same.

The best results (the smallest errors) were obtained at a rotational speed of the revolving mirror of less than one revolution per second (we used 0.8 per seconde).

The results of a series of experiments are shown in Fig. 3. Here we plotted the mean error against the total energy-flow through an opening of the screen (9) of 0.2×0.25 cm. (and observed through a lens $f=10$ cm.). We see that the lowest mean error is just over 2%. This compares favourably with the spectralpyrometer.

To compare the sensitivity of the method with that of the spectralpyrometer we worked at 6500 Å with the same HILGER monochromator. We found it was possible to work at about a ten times lower intensity. As we have no PURKINJE effect in the red, we should gain even more at shorter wavelengths as in that case one cannot measure with the spectral pyrometer. By making use of an apparatus with a larger opening one

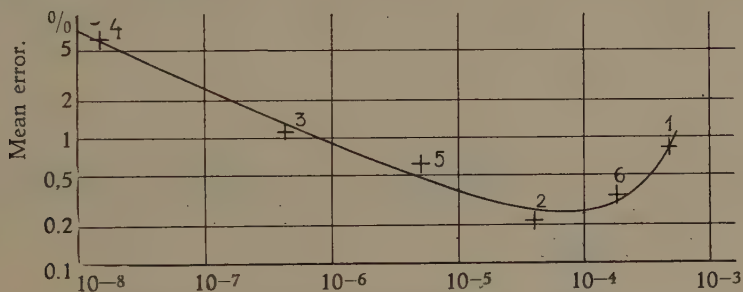


Fig. 3. Total energy flow at 5550 Å in erg/sec.

can also gain appreciably in sensitivity. One might even observe with the periphery, but that would necessitate much training.

There are two methods for observing the backslit of the monochromator. In the first place according to fig. 4a and in the second place according to

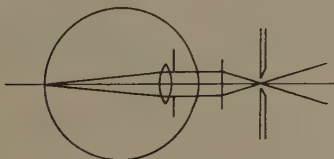


Fig. 4a. Direct view.

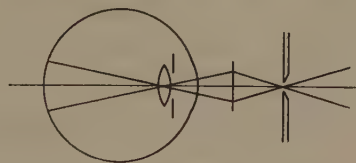


Fig. 4b. MAXWELLIan view.

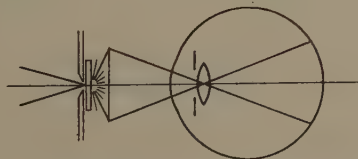


Fig. 4c. MAXWELLIan view with fluorescence.

fig. 4b. The first method will be best, as a smaller area on the retina is struck by the same amount of energy¹⁾.

¹⁾ PIPER, Zs. f. Psychol. 32, 98, 1904.

One of the primary objects of this method has been to make it possible to measure ultraviolet light monochromatically with the aid of fluorescence screens. Working according to the usual principle of observing two brightnesses simultaneously it is necessary to make a well defined image of two fields on the fluorescence screen to observe a fairly sharp borderline. These considerations (small solid angles and diffusing of the light by the fluorescence screen) make the method impractical. The difficulties mentioned are eliminated when using the method of successive contrast. In the first place it is possible to work with large solid angles, secondly the image on the screen can be concentrated and thirdly the image can be observed according to the principle of MAXWELLIAN view (see fig. 4c). In this way the maximum amount of light is observed by the eye.

Conclusion: With the method of successive contrast the same accuracy is obtained as with the spectralpyrometer, but a much higher sensitivity is obtained. It also makes possible the monochromatic measurement of ultraviolet light.

Mathematics. — *Entwicklung einer analytischen Funktion nach WHITTAKER-schen Funktionen.* Von ARTUR ERDÉLYI. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of October 31, 1936).

In vorliegender Arbeit wird die Entwicklung einer analytischen Funktion einer komplexen Veränderlichen nach konfluenten hypergeometrischen Funktionen, bzw. gemischte Entwicklungen nach solchen Funktionen und gewissen Polynomen behandelt. Die konfluenten hypergeometrischen Funktionen $M_{k,m}(z)$ ¹⁾ enthalten als Sonderfall die BESSELSchen Funktionen und können gewissermassen als Verallgemeinerung dieser Funktionenklasse aufgefasst werden. In gleicher Weise sind die zu definierenden Polynome in $\frac{1}{t}$, $A_{k,m,n}(t)$ Verallgemeinerungen der NEUMANNschen Polynome $O_n(t)$ ²⁾ bzw. der GEGENBAUERSchen Polynome $A_{n,r}(t)$ ³⁾, wie denn auch die in dieser Abhandlung bewiesenen Entwicklungen nur die

¹⁾ Wegen ihrer Definition vgl. E. T. WHITTAKER and G. N. WATSON, A course of modern analysis, 4th Ed. Cambridge 1927 insbesondere Kap. XVI. Dieses Werk wird mit M.A. zitiert.

²⁾ C. G. NEUMANN, Theorie der BESSELSchen Funktionen. Ein Analogon zur Theorie der Kugelfunktionen. Leipzig (1867); Ueber die Entwicklung beliebig gegebener Funktionen nach den BESSELSchen Funktionen. Journal für Math. 67, 310—314 (1867). Vgl. auch G. N. WATSON, A treatise on the Theory of BESSEL Functions, Cambridge (1922), insbesondere §§ 9,1—9,17. Dieses letztere Werk wird mit B.F. zitiert.

³⁾ L. GEGENBAUER, Ueber die BESSELSchen Funktionen. Sitzungsberichte der Akademie der Wissenschaften Wien (2) 74, 124—130 (1877). Vgl. auch B.F., § 9,2.

Verallgemeinerungen der bekannten NEUMANNschen Entwicklungen ⁴⁾ sind. Einige spezielle Reihen vom hier behandelten Typus kommen vor in einer früheren Arbeit des Verfassers ⁵⁾.

In weiteren Arbeiten werden die in dieser Arbeit eingeführten Polynome $A_{k,m,n}(t)$ untersucht, sowie einige Sonderfälle besprochen, die auf BESSELSche Funktionen bzw. LAGUERRESche Polynome führen.

§ 1. Entwicklung der Potenz.

Wir beginnen damit, dass wir eine beliebige Potenz von z , etwa $z^{m+\frac{1}{2}}$, in eine Reihe von der Form $\sum_{n=0}^{\infty} C_{k,m+n,n} M_{k,m+n}(z)$ entwickeln. Dabei gehen wir von einer bereits abgeleiteten Beziehung ⁶⁾ aus, die wir für unsere Zwecke nach einigen einfachen Umformungen in folgender Gestalt schreiben können:

$$z^{m+\frac{1}{2}} e^{\frac{1}{2}z} = \sum_{p=0}^{\infty} \frac{1}{p!} \frac{(k+m+\frac{1}{2})_p}{(2m+p)_p} M_{k,m+p}(z), \quad \dots \quad (1, 1)$$

gültig für jedes $z \neq 0$, jedes k und jedes m , unter der Voraussetzung dass $2m$ keine negative ganze Zahl ist. Dabei bedeutet

$$(a)_p = a(a+1) \dots (a+p-1) = \frac{\Gamma(a+p)}{\Gamma(a)}, \quad \dots \quad (1, 2)$$

Setzen wir auf der rechten Seite der Identität

$$z^{m+\frac{1}{2}} = z^{m+\frac{1}{2}} e^{\frac{1}{2}z} e^{-\frac{1}{2}z} = \sum_{s=0}^{\infty} \frac{1}{s!} \left(-\frac{1}{2}\right)^s z^{m+s+\frac{1}{2}} e^{\frac{1}{2}z}$$

für $z^{m+s+\frac{1}{2}} e^{\frac{1}{2}z}$ die aus (1, 1) durch Ersetzung von m durch $m+s$ entstehende Reihe ein, so ergibt sich

$$z^{m+\frac{1}{2}} = \sum_{s=0}^{\infty} \frac{1}{s!} \left(-\frac{1}{2}\right)^s \sum_{p=0}^{\infty} \frac{1}{p!} \frac{(k+m+s+\frac{1}{2})_p}{(2m+2s+p)_p} M_{k,m+p+s}(z).$$

In dieser Reihe setzen wir $p+s=n$ und ordnen sie entsprechend um, was wegen der absoluten Konvergenz statthaft ist, und erhalten

$$z^{m+\frac{1}{2}} = \sum_{n=0}^{\infty} M_{k,m+n}(z) \sum_{p=0}^n \frac{(k+m+n-p+\frac{1}{2})_p}{p! (n-p)! (2m+2n-p)_p} \left(-\frac{1}{2}\right)^{n-p}. \quad (1, 3)$$

⁴⁾ B.F., Kap. XVI.

⁵⁾ Ueber eine Methode zur Gewinnung von Funktionalbeziehungen zwischen WHITTAKERSchen Funktionen. Erscheint demnächst in den Monatsheften für Mathematik und Physik.

⁶⁾ I.c. Fussnote ⁵⁾ Gleichung (5, 4).

Nun folgt aus (1, 2)

$$(a)_p = (-1)^p (1-a-p)_p, \quad (1, 4)$$

daher ist

$$(k+m+n-p+\tfrac{1}{2})_p = (-1)^p (\tfrac{1}{2}-k-m-n)_p,$$

$$(2m+2n-p)_p = (-1)^p (1-2m-2n)_p$$

und

$$\frac{1}{(n-p)!} = \frac{1}{n!} \frac{n!}{(n-p)!} = \frac{(n-p+1)_p}{n!} = \frac{(-1)^p (-n)_p}{n!}.$$

Die Einsetzung der letzten drei Formeln in (1, 3) ergibt

$$z^{m+\frac{1}{2}} = \sum_{n=0}^{\infty} M_{k,m+n}(z) (-\tfrac{1}{2})^n \frac{1}{n!} \sum_{p=0}^n \frac{(-n)_p (\frac{1}{2}-k-m-n)_p}{p! (1-2m-2n)_p} 2^p$$

und daher folgenden

Satz I. Ist $2m$ keine positive oder negative ganze Zahl⁷⁾, und werden die Konstanten $C_{k,m,n}$ durch die Beziehungen erklärt

$$C_{k,m,n} = (-\tfrac{1}{2})^n \frac{1}{n!} \sum_{p=0}^n \frac{(-n)_p (\frac{1}{2}-k-m)_p}{p! (1-2m)_p} 2^p = \left\{ \begin{array}{l} \\ \\ \end{array} \right. \quad (1, 5)$$

$$= \frac{(-1)^n}{2^n \cdot n!} F(-n, \tfrac{1}{2}-k-m; 1-2m; 2),$$

so gilt für alle endlichen von Null verschiedenen Werte von z

$$z^{m+\frac{1}{2}} = \sum_{n=0}^{\infty} C_{k,m+n,n} M_{k,m+n}(z). \quad (1, 6)$$

In (1, 5) bedeutet F die gewöhnliche GAUSSsche hypergeometrische Reihe.

§ 2. Entwicklung einer analytischen Funktion im Innern eines Kreises.

Zunächst entwickeln wir die Funktion $z^{m+\frac{1}{2}}/(t-z)$ für $|z| < |t|$ in eine Reihe von der am Anfang des § 1 angegebenen Form. Diese Entwicklung stellen wir so auf, dass wir den in der zu entwickelnden Funktion vorkommenden CAUCHYSchen Kern $(t-z)^{-1}$ in die geometrische Reihe entwickeln und für jedes Glied der so erhaltenen Reihe (1, 6) verwenden. So entsteht folgende Reihe:

$$\frac{z^{m+\frac{1}{2}}}{t-z} = \sum_{r=0}^{\infty} \frac{z^{m+r+\frac{1}{2}}}{t^{r+1}} = \sum_{r=0}^{\infty} t^{-r-1} \sum_{s=0}^{\infty} C_{k,m+r+s,s} M_{k,m+r+s}(z) \quad (|z| < |t|).$$

⁷⁾ Der Wert $m=0$ ist zulässig.

Wieder setzen wir $r + s = n$ und ordnen die Doppelreihe um, was wegen der absoluten Konvergenz gestattet ist und erhalten

$$\frac{z^{m+\frac{1}{2}}}{t-z} = \sum_{n=0}^{\infty} M_{k,m+n}(z) \sum_{s=0}^n C_{k,m+n,s} t^{-n+s-1}.$$

Erklären wir also die Funktionen $A_{k,m,n}(t)$, welche Polynome $(n+1)$ -ten Grades in $\frac{1}{t}$ sind, durch die Beziehung

$$\begin{aligned} A_{k,m,n}(t) &= \frac{1}{t^{n+1}} \sum_{s=0}^n C_{k,m,s} t^s = \left(\right. \\ &= \frac{1}{t^{n+1}} \sum_{s=0}^n F\left(-s, \frac{1}{2} - k - m; 1 - 2m; 2\right) \frac{\left(-\frac{1}{2}t\right)^s}{s!}, \left. \right) \quad (2, 1) \end{aligned}$$

so ist für alle Werte von z , für welche $|z| < |t|$ ist

$$\frac{z^{m+\frac{1}{2}}}{t-z} = \sum_{n=0}^{\infty} A_{k,m+n,n}(t) M_{k,m+n}(z), \quad (2, 2)$$

vorausgesetzt dass $2m$ keine ganze Zahl ist.

Mit Hilfe von (2, 2) ist es sehr leicht, die Entwicklung einer beliebigen analytischen Funktion von z anzugeben. Es sei $f(z)$ eine innerhalb des um den Nullpunkt beschriebenen Kreises \Re und auf dessen Berandung eindeutige und reguläre Funktion der komplexen Veränderlichen z . Dann ist nach der CAUCHYSchen Formel für jeden innerhalb des Kreises \Re gelegenen Punkt z

$$f(z) z^{m+\frac{1}{2}} = \frac{1}{2\pi i} \int_{\Re} \frac{f(t) z^{m+\frac{1}{2}}}{t-z} dt.$$

Verwenden wir im Integranden (2, 2) und integrieren gliedweise, was wegen der gleichmässigen Konvergenz von (2, 2) statthaft ist, so erhalten wir

$$z^{m+\frac{1}{2}} f(z) = \sum_{n=0}^{\infty} M_{k,m+n}(z) \cdot \frac{1}{2\pi i} \int_{\Re} f(t) A_{k,m+n,n}(t) dt \quad (2, 3)$$

und hieraus folgenden

Satz II. Ist $f(z)$ eine im Innern und auf dem Rande des um den Nullpunkt beschriebenen Kreises \Re eindeutige und reguläre Funktion von z und $2m$ keine positive oder negative ganze Zahl, so ist die Funktion $z^{m+\frac{1}{2}} f(z)$ in eine nach konfluenten hypergeometrischen Funktionen fortschreitende Reihe von der Form

$$z^{m+\frac{1}{2}} f(z) = \sum_{n=0}^{\infty} a_n M_{k,m+n}(z) \quad (2, 4)$$

entwickelbar. Die Reihe konvergiert für jeden im Inneren von \Re gelegenen Punkt z und ihre Koeffizienten sind gegeben durch die Formel

$$a_n = \frac{1}{2\pi i} \int_{\Re} f(t) A_{k,m+n,n}(t) dt. \quad (2, 5)$$

Zur Berechnung der Koeffizienten kann auch die MACLAURINSche Reihe der Funktion $f(z)$ herangezogen werden. Unter den angegebenen Voraussetzungen ist im Innern und auf dem Rande von \Re

$$f(t) = \sum_{r=0}^{\infty} \frac{f^{(r)}(0)}{r!} t^r.$$

Setzen wir diesen Ausdruck, sowie (2, 1) in (2, 5) ein, so erhalten wir

$$a_n = \frac{1}{2\pi i} \int_{\Re} \sum_{r=0}^{\infty} \frac{f^{(r)}(0)}{r!} t^r \cdot \sum_{s=0}^n C_{k,m+n,s} t^{-n+s-1} dt = \sum_{s=0}^n C_{k,m+n,s} \frac{f^{(n-s)}(0)}{(n-s)!}, \quad (2, 6)$$

oder wegen (1, 5)

$$a_n = \sum_{s=0}^n \frac{(-1)^s f^{(n-s)}(0)}{2^s s! (n-s)!} F\left(-s, \frac{1}{2} - k - m - n; 1 - 2m - 2n; 2\right). \quad (2, 7)$$

§ 3. Konvergenz der Reihe $\sum_{n=0}^{\infty} a_n M_{k,m+n}(z)$.

Um über die Konvergenz der im Titel dieses § genannten unendlichen Reihe Aussagen machen zu können, untersuchen wir zunächst das asymptotische Verhalten der Funktion $M_{k,m}(z)$ für grosse positive Werte von $\Re m$. Die der Differentialgleichung

$$\frac{d^2 y}{dz^2} + \left(-\frac{1}{4} + \frac{k}{z} + \frac{\frac{1}{4} - m^2}{z^2}\right) y = 0 \quad (3, 1)$$

genügende Funktion $M_{k,m}(z)$ weist in der Umgebung des Nullpunktes eine Reihenentwicklung von der Form

$$M_{k,m}(z) = z^{m+\frac{1}{2}} (1 + c_1 z + c_2 z^2 + \dots) \quad (3, 2)$$

auf⁸⁾. Gehen wir mit dem Ansatz (3, 2) in (3, 1) ein und vergleichen die Koeffizienten gleich hoher Potenzen von z , so erhalten wir mit den Festsetzungen

$$c_{-1} = 0, \quad c_0 = 1$$

⁸⁾ M.A., § 16,1.

folgende Rücklaufformel für die Koeffizienten c_r ⁹⁾

$$[(m+r+\frac{1}{2})(m+r-\frac{1}{2})+\frac{1}{4}-m^2]c_r+k c_{r-1}-\frac{1}{4}c_{r-2}=0 \quad (r=1, 2, \dots),$$

aus der sich

$$c_r = \frac{\frac{1}{4}c_{r-2} - k c_{r-1}}{r(2m+r)} \cdot \dots \cdot \dots \quad (3, 3)$$

ergibt. Daraus folgt für sehr grosse positive Werte von $\Re m$

$$c_r = O\left(\frac{1}{[\Re m]^{\frac{1}{2}+\frac{1}{2}r}}\right) \quad (r=1, 2, \dots) \cdot \dots \cdot \dots \quad (3, 4)$$

gleichmässig in r , also

$$M_{k,m}(z) = z^{m+\frac{1}{2}} \left[1 + O\left(\frac{1}{\Re m}\right) \right] \cdot \dots \cdot \dots \quad (3, 5)$$

Nachdem wir das asymptotische Verhalten der konfluenten hypergeometrischen Funktion in dem uns interessierenden Falle kennengelernt haben, können wir sofort die Frage nach der Konvergenz der im Titel genannten Reihe beantworten. Aus (3, 5) folgt insbesondere, dass für hinreichend grosse positiv ganzzahlige Werte von n stets

$$M_{k,m+n}(z) = z^{m+n+\frac{1}{2}} \left[1 + O\left(\frac{1}{n}\right) \right] \cdot \dots \cdot \dots \quad (3, 6)$$

ist. Daraus geht hervor, dass die Reihe $\sum_{n=0}^{\infty} a_n M_{k,m+n}(z)$ genau wie die ihr „zugeordnete Potenzreihe“ $\sum_{n=0}^{\infty} a_n z^n$ einen Konvergenzkreis besitzt, dessen Halbmesser durch die Formel

$$R = \frac{1}{\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}} \cdot \dots \cdot \dots \quad (3, 7)$$

gegeben ist; im Innern des Konvergenzkreises konvergiert sie absolut und gleichmässig und divergiert in jedem Punkte ausserhalb des Konvergenzkreises. Genau so, wie bei den Potenzreihen zeigt man, dass auf dem Rande des Konvergenzkreises mindestens eine Singularität der durch die Reihe dargestellten innerhalb des Konvergenzkreises mit Ausnahme des Punktes $z=0$ überall regulären Funktion liegen muss. Ferner kann man genau so,

⁹⁾ Da für den hier verfolgten Zweck nur die Grössenordnung der Koeffizienten von Belang ist, begnügen wir uns an dieser Stelle mit der Aufstellung der Rekursionsformeln und verzichten auf die explizite Ausrechnung der Koeffizienten, die übrigens durch hypergeometrische (JACOBISCHE) Polynome dargestellt werden können.

wie WATSON es im Falle der NEUMANNschen Reihen getan hat¹⁰⁾ zeigen, dass die im Titel genannte Reihe höchstens dieselben Singularitäten haben kann, wie die ihr zugeordnete Potenzreihe.

§ 4. Entwicklungen in einem Ringgebiet.

Es seien \Re bzw. r zwei konzentrische Kreise mit den Halbmessern R bzw. r , deren gemeinsamer Mittelpunkt der Punkt $z = 0$ ist. Dann ist für

$$r < |z| < R. \quad (4, 1)$$

nach (2, 2) für t auf \Re

$$\frac{1}{t-z} = z^{-m-\frac{1}{2}} \sum_{n=0}^{\infty} A_{k,m+n,n}(t) M_{k,m+n}(z), \quad (4, 2)$$

während für t auf r nach derselben Gleichung wegen (4, 1)

$$\frac{1}{t-z} = -t^{-m-\frac{1}{2}} \sum_{n=0}^{\infty} A_{k,m+n,n}(z) M_{k,m+n}(t). \quad (4, 3)$$

ist.

Ist $f(z)$ eine im Bereich $r \leq |z| \leq R$ eindeutige und reguläre Funktion von z , so ist bekanntlich für $r < |z| < R$

$$f(z) = \frac{1}{2\pi i} \int_{\Re} \frac{f(t) dt}{t-z} - \frac{1}{2\pi i} \int_r \frac{f(t) dt}{t-z}.$$

Da für jeden inneren Punkt dieses Gebietes (4, 1) erfüllt ist, so können wir im ersten Integral (4, 2), im zweiten (4, 3) verwenden und gliedweise integrieren, wodurch sich folgende Beziehung ergibt:

$$\left. \begin{aligned} f(z) = z^{-m-\frac{1}{2}} \sum_{n=0}^{\infty} M_{k,m+n}(z) \frac{1}{2\pi i} \int_{\Re} f(t) A_{k,m+n,n}(t) dt \\ + \sum_{n=0}^{\infty} A_{k,m+n,n}(z) \frac{1}{2\pi i} \int_r f(t) t^{-m-\frac{1}{2}} M_{k,m+n}(t) dt. \end{aligned} \right\} \quad (4, 4)$$

Daher besteht folgender

Satz III. Ist $f(z)$ eine im Innern und auf dem Rande des durch die beiden Kreise \Re und r begrenzten Ringbereiches eindeutige und reguläre Funktion von z , so gestattet sie, vorausgesetzt, dass $2m$ keine positive oder

¹⁰⁾ B.F., § 16,2.

negative ganze Zahl ist, eine in allen inneren Punkten dieses Gebietes konvergente Reihentwicklung der Gestalt

$$f(z) = z^{-m-\frac{1}{2}} \sum_{n=0}^{\infty} a_n M_{k,m+n}(z) + \sum_{n=0}^{\infty} b_n A_{k,m+n,n}(z), \quad (4, 5)$$

deren Koeffizienten auf folgende Weise berechnet werden können:

$$a_n = \frac{1}{2\pi i} \int_{\mathfrak{R}} f(t) A_{k,m+n,n}(t) dt, \quad b_n = \frac{1}{2\pi i} \int_{\mathfrak{r}} f(t) t^{-m-\frac{1}{2}} M_{k,m+n}(t) dt. \quad (4, 6)$$

Ähnlich wie in § 2 können die Koeffizienten a_n und b_n durch die Koeffizienten der LAURENTSchen Entwicklung von $f(z)$ ausgedrückt werden. Ist insbesondere $f(z)$ noch überdies im Innern von \mathfrak{r} überall eindeutig und regulär, so verschwinden alle Koeffizienten b_n ($n=0, 1, 2, \dots$) und die Entwicklung (4, 5) reduziert sich auf (2, 4).

*Mathematisches Seminar
der Deutschen Technischen Hochschule.*

Brünn, Juni 1936.

Chemistry. — *Recherche quantitative sur l'échange d'ions produit par l'addition, au sol négatif de AgI, de nitrates de thorium, d'hexol et de cérium.* Par J. GILLIS et J. EECKHOUT. (Communicated by Prof. H. R. KRUYT.)

(Communicated at the meeting of October 31, 1936).

I. Introduction.

Les travaux de H. G. BUNGENBERG DE JONG ont révélé que le nitrate de thorium est en état de produire la coacervation lorsqu'on l'ajoute à la gomme arabique p.e. Cette *auto-coacervation complexe*, produite par des ions polyvalents de signe contraire à celui du sol, s'expliquerait par l'apparition, sur les micelles du même colloïde, de charges de signes opposés.

D'après BUNGENBERG DE JONG¹⁾, l'ion polyvalent de signe contraire à celui du sol serait à même de créer des centres de signe opposé par le fait qu'une partie seulement de son champ électrique serait neutralisée à la

¹⁾ H. G. BUNGENBERG DE JONG et J. LENS: Ueber äquivalent Entladung und Umladung bei lyophilen Solen. *Bioch. Zeitschr.* **235**, p. 174 (1931).

surface de la micelle, le reste de son champ restant orienté vers l'extérieur.

Cette théorie capable de rendre un compte satisfaisant du phénomène d'auto-coacervation complexe, n'avait pas encore été contrôlée dans le cas des colloïdes lyophobes et c'est pourquoi le Prof. H. R. KRUYT nous proposa, pendant son séjour à Gand en qualité de titulaire de la chaire-FRANCQUI, d'en entreprendre le contrôle expérimental, p.e. vis-à-vis du sol négatif, bien défini, d'iodure d'argent.

Le problème pouvait se poser de la manière suivante (voir fig. 1).

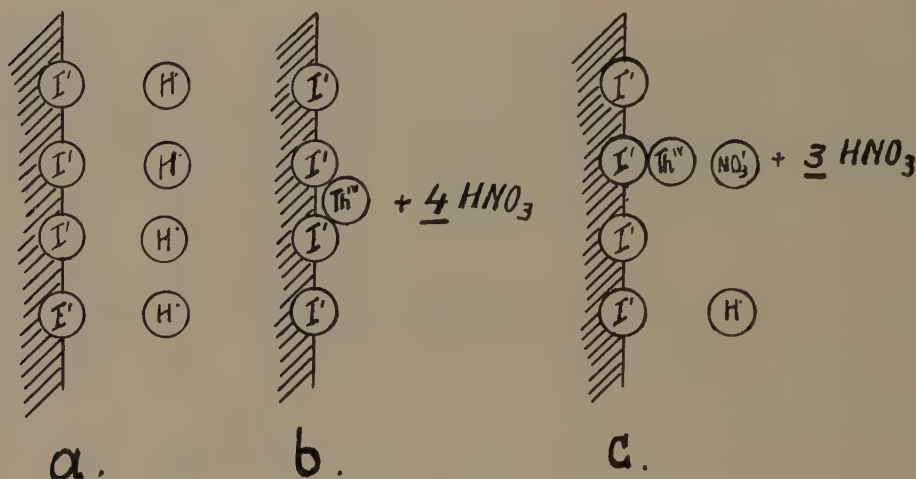


Fig. 1.

Admettons d'une manière toute schématique que la fig. 1a représente une portion de la couche électrique double du sol dialysé de AgI, avec en regard les uns des autres les ions d'iode fixes et les ions d'hydrogène mobiles. En ajoutant du nitrate de thorium jusqu'à flocculation p.e., nous pouvons nous figurer, comme le traduit la fig. 1b, que, pour chaque ion de thorium entraîné dans le floculat, nous retrouverons quatre ions d'hydrogène et quatre ions NO_3 dans la solution coexistante. Dans ce cas, où la théorie susdite ne serait d'ailleurs pas vérifiée, tous les ions NO_3 ajoutés devraient donc se retrouver quantitativement dans le liquide en équilibre avec le précipité.

Dans le cas où la théorie se vérifierait au contraire, il faudrait qu'il y ait toujours moins de quatre ions d'hydrogène échangés pour chaque ion de thorium entraîné par le précipité et il faudrait également qu'une partie des ions NO_3 ajoutés soit entraînée par le précipité. C'est ce que nous avons admis dans la fig. 1c, où trois molécules de HNO_3 parviendraient dans la solution, tandis que un ion NO_3 passerait dans le floculat.

Envisagé de cette manière le problème devait donc pouvoir se résoudre par voie analytique; à première vue nous pouvions croire qu'il suffirait de doser les ions H et les ions NO_3 échangés, après l'addition de quantités connues de nitrate d'un ion polyvalent.

II. *Partie expérimentale.*

La concentration des ions d'hydrogène fut établie, après floculation et centrifugation, dans le liquide clair décanté, à l'aide d'une électrode de verre et d'un potentiomètre à lampe électrométrique („Cambridge Valve-potentiometer”).

La quantité totale des ions d'hydrogène du sol fut établie par titrage potentiométrique dans une atmosphère inerte à l'aide de NaOH (± 0.001 n.).

Le dosage des ions NO_3 a été effectué par la méthode polarographique de HEYROVSKÝ¹⁾, qui permet encore de les évaluer avec une approximation suffisante jusqu'aux très faibles concentrations de 10^{-6} mole par litre.

Afin d'obtenir des variations de concentration aussi grandes que possible des ions échangés et de permettre un dosage suffisamment rigoureux des concentrations minimales de ces ions, nous avons tout intérêt à opérer sur des sols d'iodure d'argent aussi concentrés que possible. Nous les avons obtenus par électrodialyse et électrodécantation à des concentrations de l'ordre de 300 millimoles par litre. D'autre part nous avons poursuivi l'électrodialyse non seulement jusqu'à réaction négative à la diphenylamine, qui laissait encore 10^{-4} iongramme NO_3 au litre dans le sol, mais encore jusqu'à réaction négative des ions NO_3 au polarographe, ce qui excluait ces ions jusqu'à 10^{-6} iongramme au moins au litre.

Nos recherches nous montrèrent bientôt qu'il était également nécessaire de doser les ions métalliques présents dans le liquide en équilibre avec le précipité. Cela nous a amené à doser de petites quantités de thorium, d'hexol et de cérium dans le décantat et ultérieurement à doser de minimales quantités de plomb et de zinc dont nous avons pu reconnaître la présence dans nos sols.

Pour chaque expérience nous avons toujours mélangé $a \text{ cm}^3$ de sol à $a \text{ cm}^3$ de solution de nitrate dans un tube à centrifugation. Dans un deuxième tube nous avons mélangé $a \text{ cm}^3$ d'eau à $a \text{ cm}^3$ de la même solution de nitrate. Les deux tubes ont toujours subi un traitement parallèle en ce qui concerne les dosages. Pour le premier tube les analyses ont eu lieu sur le décantat, après avoir centrifugé le précipité.

III. *Résultats obtenus.*

Nous avons choisi les nitrates de thorium, d'hexol et de cérium pour effectuer nos expériences; nous pourrions étudier respectivement ainsi l'action d'ions positifs tétra-, hexa- et trivalents sur le sol négatif d'iodure d'argent.

¹⁾ W. BÖTTGER. *Physikalische Methoden der Analytischen Chemie*. II. p. 260. Leipzig 1936.

Les résultats obtenus ont été réunis dans la table suivante: ¹⁾

TABLE I.

Sol	Electro-lyte	Concentration finale de l'électrolyte	Ions H échangés	Ions NO ₃ dans le décantat.	Ions NO ₃ dans le floculat	Cations mét. en excès da le décantat
I { Conc. Ag I: 298 mM. p _H : 3.91	Th(NO ₃) ₄	5 . 10 ⁻⁴ E.	0.60 . 10 ⁻⁴ E.	4.77 . 10 ⁻⁴ E.	0.23 . 10 ⁻⁴ E.	2.15 . 10 ⁻⁴
		6 . 10 ⁻⁴ E.	0.65 . 10 ⁻⁴ E.	5.74 . 10 ⁻⁴ E.	0.26 . 10 ⁻⁴ E.	2.92 . 10 ⁻⁴
II { Conc. Ag I: 366 mM. p _H : 3.93	Hexol-nitrate	5 . 10 ⁻⁴ E.	0.12 . 10 ⁻⁴ E.*	4.60 . 10 ⁻⁴ E.	0.40 . 10 ⁻⁴ E.	0.90 . 10 ⁻⁴
		6 . 10 ⁻⁴ E.	0.04 . 10 ⁻⁴ E.*	5.55 . 10 ⁻⁴ E.	0.45 . 10 ⁻⁴ E.	2.00 . 10 ⁻⁴
III { Conc. Ag I: 286 mM. p _H : 3.89	Ce(NO ₃) ₃	5.05 . 10 ⁻⁴ E.	0.49 . 10 ⁻⁴ E.	5.02 . 10 ⁻⁴ E.	0	2.82 . 10 ⁻⁴
		6.02 . 10 ⁻⁴ E.	0.60 . 10 ⁻⁴ E.	6.00 . 10 ⁻⁴ E.	0	3.82 . 10 ⁻⁴

L'examen de ces données révèle de suite que, pour les nitrates de thorium et d'hexol, les ions NO₃ ne se retrouvent pas d'une manière quantitative dans le décantat; les ions manquants ne peuvent se trouver ailleurs que dans le floculat. C'est cette partie que nous avons indiquée dans la sixième colonne. Pour le nitrate de cerium, au contraire, tous les ions NO₃ se retrouvent dans le décantat.

D'autre part nous voyons que pour les trois électrolytes envisagés il y a plus de cations de thorium, d'hexol ou de cérium entraînés dans le floculat que ne correspond aux ions d'hydrogène échangés, augmentés des équivalents d'ions NO₃ entraînés dans le précipité. Il fallait donc bien admettre que d'autres cations que les ions d'hydrogène avaient été échangés.

Nous avons d'abord soupçonné la présence d'ions de potassium dans le décantat, mais nous ne les y avons pas reconnus par les réactions les plus sensibles. Toutefois les polarogrammes du décantat révélaient deux „ondes” que nous sommes parvenus à identifier et qui révélaient la présence de Pb et de Zn, qui devaient donc avoir été échangés également par les ions positifs polyvalents. Nous avons pu doser ces ions de Pb et de Zn par voie polarographique et la quantité trouvée ainsi compensait la presque totalité des cations déficients.

Nous avons reconnu alors que notre eau distillée ayant servi au lavage prolongé (quatre à cinq semaines) des sols pendant l'électrodialyse, renfermait en effet des traces de Pb et de Zn de l'ordre de 1 à 2 × 10⁻⁷ E au litre. Vue la grande quantité d'eau employée, ces ions auront pu s'accumuler dans le sol, dont d'ailleurs l'acidité diminuait lentement au cours de l'élec-

¹⁾ La lettre E désigne partout des équivalentsgrammes. L'astérisque attire l'attention sur des valeurs anormales des ions d'hydrogène échangés, dans les expériences où l'hexol intervient. Elles sont dues à la décomposition de l'hexol lui-même qui fixe alors des ions d'hydrogène ainsi que des expériences entreprises à cet effet nous l'ont montré.

trodialyse. H. R. KRUYT et E. J. W. VERWEY¹⁾ avaient déjà montré que de pareilles impuretés ne se laissent pas éliminer par dialyse prolongée.

IV. *Conclusions.*

1o. La théorie de BUNGENBERG DE JONG se confirme également pour le sol négatif d'iodure d'argent, tout au moins en ce qui concerne les ions tétravalent et hexavalent de thorium et d'hexol. Une partie des ions NO_3 est entraînée notamment par le précipité. Cette partie est toutefois plus faible que pour les sols lyophiles.

L'ion trivalent de cérium au contraire n'est déjà plus en état de provoquer de façon appréciable l'apparition de centres positifs sur le sol de AgI .

2o. Les ions polyvalents, pour produire la floculation du sol, ne passent pas d'une manière quantitative dans la partie fixe de la couche double. La dernière colonne de la table 1 montre qu'il faut chaque fois un grand excès de cation polyvalent, qui reste dans le floculat, pour que la floculation du sol puisse avoir lieu.

3o. En comparant la quantité d'ions d'hydrogène échangée, après floculation par les ions polyvalents, à la quantité totale des ions d'hydrogène déduite des courbes de titrage potentiométriques, nous avons constaté que les ions d'hydrogène ne sont pas échangés de manière quantitative, mais qu'il s'établit un équilibre entre ces ions.

Nos résultats seront publiés in extenso à un autre endroit. Dès maintenant nous ne pouvons manquer d'adresser à Monsieur H. R. KRUYT tous nos remerciements pour l'intérêt qu'il n'a cessé de témoigner pour nos recherches.

Laboratoire de Chimie analytique de l'Université.

Gand, Nov. 1936.

¹⁾ E. J. W. VERWEY et H. R. KRUYT. Zur Kenntnis der elektrischen Doppelschicht bei Kolloiden. IV. Zeits. f. Phys. Chem. Abt. A. **167**, 325 (1934).

Chemistry. — *Further examples of unmixing in aqueous salt solutions.*
By H. G. BUNGENBERG DE JONG and L. TEUNISSEN-VAN ZIJP.
(Communicated by Prof. H. R. KRUYT).

(Communicated at the meeting of October 31, 1936).

In a previous publication¹⁾ examples were given of unmixing in aqueous salt solutions and the significance of these phenomena for the theory of the complex- and auto-complex coacervation was discussed.

¹⁾ H. R. KRUYT and H. G. BUNGENBERG DE JONG, These Proc. **38**, 714 (1935).

Unmixing in aqueous electrolyte solutions is by no means a common phenomenon. In the literature BAKHUIS ROOZEBOOM²⁾ mentions some cases, viz.:

the Cu salt of valerianic acid and caproic acid
Pb-butyrate and Mg-undecylate

These cases are so little stable that they cannot be studied any further; the salts crystallize quickly. A better example is the system water- K-salt of trinitro-oxyphenyl-methylnitramine which has a metastable unmixing-area and of which ROOZEBOOM could make analyses of upper and lower layer.

In the microchemical literature it is often seen that alkaloids unmix with various reagents. A systematic investigation has not yet been made, but we have already found that unmixing sets in when a solution of novocain chloride is mixed with a solution of:

$K_3Co(CN)_6$
 $K_3Fe(CN)_6$
 NH_4 -heptamolybdate
Na-picrate
 K_2HgI_4

It seems desirable to be acquainted with as many examples as possible in order that we may know in which combinations of two salts unmixing may be expected during the double transmutation. Here we communicate our observations in the combinations:

Na-salt of carboxylic acid + Me-chloride or nitrate where
Me = anorganic cation.

The method of investigation was the same as that described in the previous communication:

On an object-glass are placed side by side a drop of a solution of one of the salts of group A and a drop of a solution of one of the salts of group B. The two drops are then covered with a cover-glass and the contact zone is microscopically examined. If necessary, the preparation is heated for a short while and once more studied microscopically. In this way all mutual combinations of the salts of group A with those of group B have been examined.

From a negative result it should not always be concluded that unmixing is not possible; occasionally the solutions of the organic salts were not stronger than 0.1 N, so that it is not impossible that in higher concentrations the salts may unmix.

²⁾ H. W. BAKHUIS ROOZEBOOM, Rec. Trav. Chim. des Pays Bas 8, 257 (1899).

- A. Nitrates of Ce, La, UO_2 , Pb, Cu, Cd, Zn, Mn, Co, Ni, Ag;
Chlorides of $\text{Co}(\text{NH})_6$, Mg, Ca, Sr, Ba, Li, Na, K, NH_4 .
- B. Sodium salts of:
Valerianic acid, caproic acid, caprylic acid, capric acid;
monochloroacetic acid, trichloroacetic acid;
oxalic acid, malonic acid, succinic acid, glutaric acid, adipic acid,
suberic acid, azelaic acid, sebacic acid;
fumaric acid, maleic acid, malic acid, tartaric acid, citric acid, asparaginic
acid;
diethylbarbituric acid;
benzoic acid and its derivatives: p-F, p-Cl, p-Br, p-I, o-oxy, p-oxy,
p-amido, p-methoxy-benzoic acids;
phenylacetic acid, phenylpropionic acid, mandelic acid, cinnamic acid;
phthalic acid, terephthalic acid;
phenolsulphonic acid, α -naphthalinsulphonic acid, β -naphthalinsulphonic
acid, naphtholdisulphonic acid.

In the following table the combinations have been recorded in which unmixing occurs with certainty. In many of these cases the liquid state of the precipitate cannot be stated for certain at room-temperature, but only becomes apparent on heating.

Na valerianate	+ $\text{Pb}(\text{NO}_3)_2$: large drops
"	+ $\text{Zn}(\text{NO}_3)_2$: drops and crystals
Na caproate	+ $\text{Ce}(\text{NO}_3)_2$: drops on heating
"	+ $\text{UO}_2(\text{NO}_3)_2$: " "
"	+ $\text{Pb}(\text{NO}_3)_2$: " "
Na caprylate	+ $\text{Co}(\text{NH}_3)_6\text{Cl}_3$: small drops
"	+ $\text{UO}_2(\text{NO}_3)_2$: drops
"	+ $\text{Cd}(\text{NO}_3)_2$: "
"	+ $\text{Mn}(\text{NO}_3)_2$: "
"	+ $\text{Co}(\text{NO}_3)_2$: " on heating
"	+ $\text{Ni}(\text{NO}_3)_2$: " "
Na caprate	+ $\text{UO}_2(\text{NO}_3)_2$: " in the cold
Na succinate	+ $\text{Pb}(\text{NO}_3)_2$: particularly in the heat very beautiful
Na glutarate	+ $\text{UO}_2(\text{NO}_3)_2$: drops on heating
"	+ $\text{Pb}(\text{NO}_3)_2$: " in the cold
Na adipate	+ $\text{Pb}(\text{NO}_3)_2$: unmixing
Na sebacate	+ $\text{UO}_2(\text{NO}_3)_2$: on heating very beautiful unmixing
Na malate	+ $\text{Pb}(\text{NO}_3)_2$: on heating
"	+ $\text{Cd}(\text{NO}_3)_2$: " beautiful
Na tartrate	+ $\text{Cd}(\text{NO}_3)_2$: " "

Na asparaginate	+ Pb(NO ₃) ₂	: beautiful unmixing
"	+ UO ₂ (NO ₃) ₂	: in the heat liquid
Na benzoate	+ Cd(NO ₃) ₂	: in the heat very beautiful
Na salt of p-toluic acid	+ Pb(NO ₃) ₂	: on heating, not beautiful
" o- "	+ Pb(NO ₃) ₂	: " "
" phenylacetic "	+ UO ₂ (NO ₃) ₂	: " "
" " "	+ Cd(NO ₃) ₂	: beautiful on heating
" phenylpropionic "	+ UO ₂ (NO ₃) ₂	: unmixing on heating
" " "	+ Cd(NO ₃) ₂	: beautiful on heating

In the 800 examined combinations of the 20 salts of group A with the 40 salts of group B we could only state unmixing with certainty in 29 cases.

The number of cases stated with each of the cations amounted to: Pb⁺⁺ = 9, UO₂⁺⁺ = 8, Cd⁺⁺ = 6, Ce⁺⁺⁺ = 1, Co(NH₃)₆⁺⁺⁺ = 1, Zn⁺⁺ = 1, Mn⁺⁺ = 1, Ni⁺⁺ = 1, Co⁺⁺ = 1.

Evidently in particular the Pb, UO₂ and Cd salts with the Sodium salts of carboxylic acids examined here are in a favourite position with regard to an unmixing at the double transmutation.

Botany. — *Concerning the influence of temperature treatment on the carbohydrate metabolism, the respiration and the morphological development of the tulip.* III. By L. ALGERA. (Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of October 31, 1936).

DISCUSSION OF THE RESULTS.

The respiration and the carbohydrate metabolism.

Table III and figures 6 and 8 of the preceding publication (1) show that the respiration also after being calculated on the basis of 20° C, is dissimilar in the various lots. Is this to be explained by the assumption that the applied temperature influences the forming or activation of the respiratory enzymes or can the differences discovered be attributed to the difference in the sugar percentage. It has been proved that the concentration of the reducing sugars is little changed by the temperature treatment. It is more obvious to ascertain whether there exists a relationship between the non-reducing sugars and the respiration.

If the intensity of the respiration was exclusively controlled by the percentage of these sugars, then there would have to be a strict proportion between them. The quotient:

$$\frac{\text{carbon dioxide emission (oxygen consumption) per K.G. dry weight/hour}}{\text{quantity non-reducing sugars per K.G. dry weight}}$$

should then on the different dates not only be continually equal in the same lot but also in all the lots individually.

Table III (1) and the figures 1 and 2 show how far this is correct. From 26th July till 17th August the value of the quotient declines. After the division the lines of LN 20-20-op, LN 20-13-op and LN 20-9-op at first drop a little and then gradually rise. The deviations of the horizontal line are however slight so that in this period there is a fairly good proportion between the respiration and the non-reducing sugars. With LN 20-5/4-op the value of the quotient continues declining up to planting, and at planting it is then considerably lower. With LN 20-17-op it continually keeps on rising up to planting. A month after the planting the differences have chiefly disappeared again.

With the lots, which were planted at 13° C., the value of the quotient increases rapidly at first after which a slower increase ensues and then drops again. The proportion is not so clear here.

As the non-reducing sugars chiefly consist of sucrose, the respiration was also calculated on the basis of this sugar. A proportion is however hardly perceptible here.

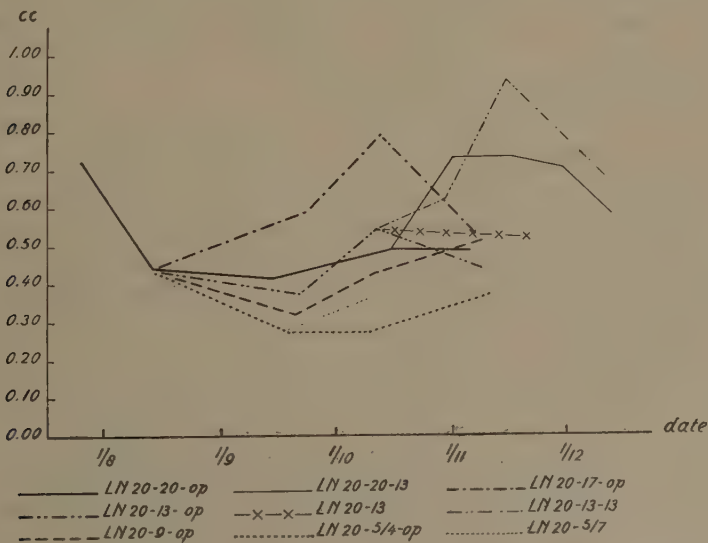


Fig. 1. Le Nôtre. Season 1934—1935. Carbon dioxide liberation (calculated on the basis of 20° C.) divided by the non-reducing sugars.

From the proportion found between the intensity of the respiration and the concentration of the non-reducing sugars, we can deduce something concerning the way in which the sugars discussed are formed.

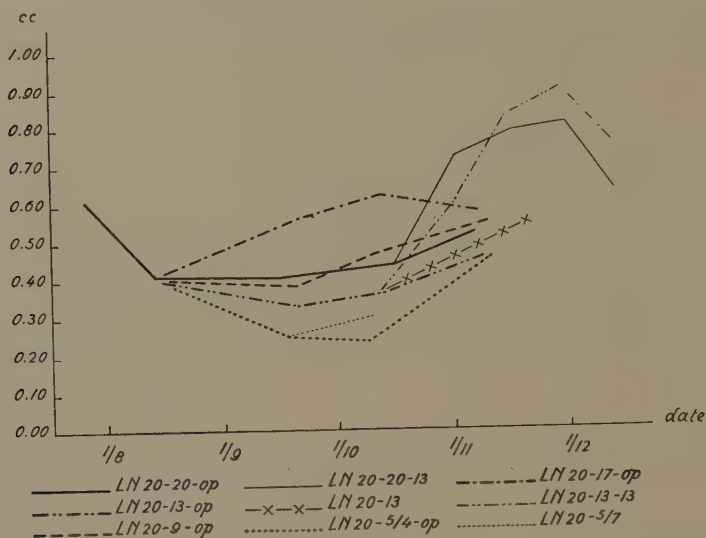


Fig. 2. Le Nôtre. Season 1934—1935. Oxygen consumption (calculated on the basis of 20° C.) divided by the non-reducing sugars.

Three views are possible with regard to their way of formation:

1°. The starch-decomposition takes place according to the scheme:
starch \rightarrow non-reducing sugars \rightarrow reducing sugars.

2°. The reducing sugars are formed in another way than as indicated in 1°; the non-reducing sugars are secondarily formed out of the reducing sugars.

3°. Both possibilities are combined.

As the concentration of the reducing sugars is very small up to some time after the planting and decreases rather than increases, the supposition expressed in 2° leads to the presumption that the reducing sugars, which arise through unknown intermediate reactions are partly respired, and for the rest are converted into non-reducing sugars. According to this view, a proportion between the non-reducing sugars and the respiration is not very explicable. A proportion with the reducing sugars would then be more conceivable.

It is therefore more probable that the reducing sugars are formed via the non-reducing sugars. The greater the concentration of the latter sugars, the quicker the reducing sugars will be formed from them under otherwise equal conditions. Notwithstanding this fact, their concentration between 20° and 9° C. is however the same and at 5 (4)° C. only a little higher. A simple explanation for this equality is the assumption, that the reaction: non-reducing sugars \rightarrow reducing sugars is so tardy that the

reducing sugars formed are immediately respired. In these circumstances the respiratory process is thus limited by the quantity of respirable sugars available and its rate depends on the intensity with which these sugars are formed. As the rate of this formation calculated on the basis of equal temperature depends on the concentration of the non-reducing sugars, it is clear why the respiration, calculated on the basis of 20°C ., in such a great measure is proportionate to this concentration. The low value of the quotient at $5\text{ (4)}^{\circ}\text{C}$. perhaps shows that the concentration is so high here that it no longer acts as a limiting factor for the respiration. Through this the percentage of reducing sugars too, may be somewhat higher. It is not yet explicable why the quotient with LN 20-17-op before the planting is so high.

As a matter of fact the proportion discovered is not absolute; just before and after planting especially at 13°C ., the respiration increases more rapidly than in accordance with the concentration of the non-reducing sugars. This can be attributed to an acceleration of the reaction: non-reducing sugars \rightarrow reducing sugars, e.g. by an increase or activating of the enzymes which control the rate of this process. There however exists a possibility too, that as the season advances, continually more reducing sugars are being formed by another way than via the non-reducing sugars. In other words the case put in 3° is becoming more and more actual. When in this way reducing sugars are formed faster and faster a point of time comes after which the respiratory enzymes are no longer able to utilize all of these sugars and from now on the percentage of reducing sugars begins to increase.

After the intensity of the respiration has been largely deduced from the quantity of the non-reducing sugars and the small percentage of reducing sugars has been attributed to respiration, the question is raised to what factors the increase of the non-reducing sugars in the cooled bulbs are due.

The temperature treatment and the non-reducing sugars.

Also on discussing the relationship between temperature treatment and the amount of non-reducing sugars, the respiration has to be considered. The increase of these sugars at a lower temperature can be explained in two ways. It can be a result of a quicker starch decomposition but also of a slower sugar utilization, so in our case of a slower respiration.

According to MÜLLER—THURGAU (3) the latter possibility has been realized with the potato.

The quantity of starch which at a certain date since 26th July has been converted into sugars, is equal to the increase of the sugars existing, increased with the quantity which till that time has been respired.

In table III (1) and fig. 3 the result of this calculation is summed up on a basis of 1 K.G. dry weight. The starch decomposition is quicker in

proportion to the temperature being lower, except that it is slower at 9° than at 13° C. In the period from 17th August till 9th October, at $5\ (4)^{\circ}$ C. about twice as much starch is converted than at 20° C.

SNELL (4) also arrives at a similar conclusion with the potato. At the same time he refers to HOPKINS and SCHANDER, who also combat the view of MÜLLER—THURGAU.

The differences largely disappear after the planting. The bulbs which have received a warmer treatment overtake the more cooled bulbs through a more rapid starch decomposition.

Shortly after the planting too, the lowering of temperature promotes the sugar formation. This is evident if LN 20—13—op is compared with LN 20—13—13 and LN 20—20—op with LN 20—20—13. In both the

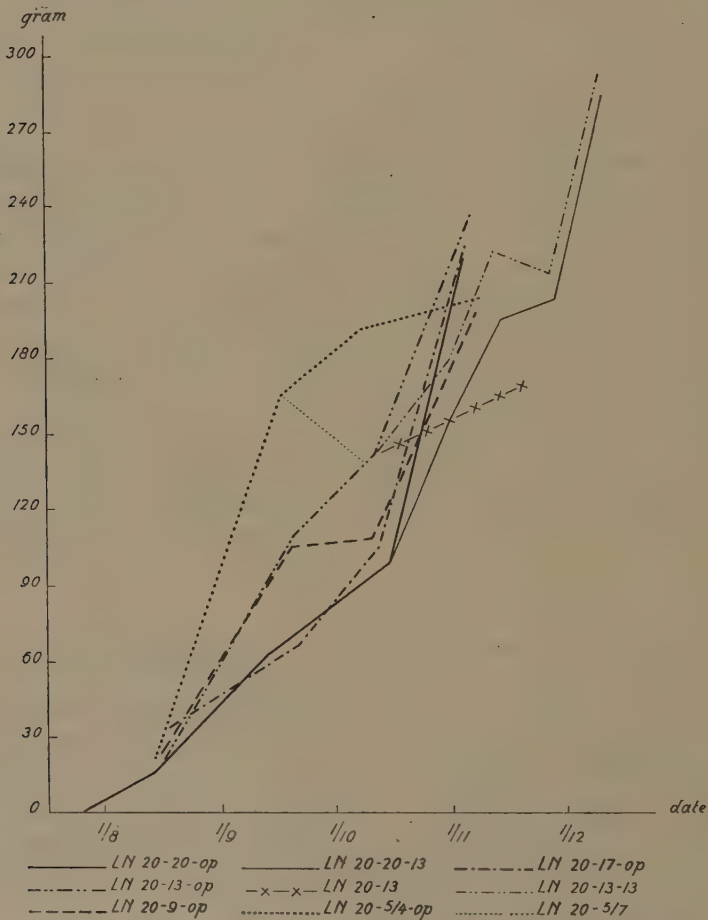


Fig. 3. Le Nôtre. Season 1934—1935. Sugar increase since 26th July + respired food per K.G. dry weight (in gr.).

cases the sugar formation at 13° C. is less rapid than in the open, although the average temperature here was 9° C. The difference between

LN 20—13—13 and LN 20—13 shows that the planting is also of influence.

In general a chemical conversion takes place quicker at a higher than at a lower temperature. This gives rise to the assumption that through the effect of the temperature something changes in the bulb, through which just the reverse is brought about.

So in this way cooling would for instance be able to effect an increase or activation of the enzymes which control the reaction starch→sugars. By greater enzyme activity the retarding effect of a lower temperature would be more than compensated.

This explanation does not seem to me to be correct, for if this were the case, then the bulbs at planting would have to contain more enzyme in proportion to the cooling having been cooler. As all the lots planted in the open have been subjected here to the same temperature conditions, then the cooled bulbs should form sugar at the greatest rate. The reverse is however the case.

At planting LN 20—13—13 will contain just as many enzymes as LN 20—13—op and LN 20—20—13 as many as LN 20—20—op. The warmest planted bulbs should then form sugar quickest. Likewise LN 20—13—13 should be more rapid than LN 20—20—13. It is rather tardier, although the difference is not very great.

LN 20—5/7 in the period from 24th September till 9th October should also be quicker than LN 20—5/4—op. Instead of increasing, a part of the sugars is even re-formed into starch.

According to my point of view, the following explanation, is at least provisionally, more satisfactory. This explanation makes use of the principle of the moving equilibrium. This principle says, as far as it applies to the temperature, that with a process which takes place under evolution of heat, the entire quantity of substance formed increases by a lowering of the temperature. In other words the process only comes to an equilibrium at a higher concentration of the substance formed.

The analyses indeed show an influence of the temperature on the equilibrium concentration of the non-reducing sugars (Table II and fig. 1 of the preceding publication).

The bulbs of the season of 1934—1935 were lifted on 20th July 1934. Round about this date the outside temperature was about 20° C. From 20th July till 24th July the bulbs were in an unwarmed bulb house, where the temperature will also have been in the neighbourhood of 20° C. On the 24th of July the bulbs were transferred to a room where the temperature was regulated exactly at 20° C. On 26th July they had therefore been exposed to 20° C. for some time. The fact that on 17th August the percentage of non-reducing sugars is equal to that on 26th July indicates that an equilibrium has been reached. Only such a quantity of sugar is formed as is absorbed by the respiration. If the parcel is divided on 17th August and the bulbs transferred to 5, 9, 13 and 17° C. resp. remain at 20° C., the latter will

be near to their equilibrium. The others are further removed from it, in proportion to their having been put in a lower temperature and as a reaction passes quicker in proportion to its being removed further from its equilibrium, the sugar formation, for this reason will take place more rapidly as the temperature is lower. This tendency is apparently stronger than the retarding effect of the fall of temperature.

It is not known whether the concentrations at planting on 9th October are equilibrium concentrations. The differences would perhaps have been still greater on the treatment being prolonged further.

The fact that all lots after having been planted in the open a month contain about the same amount of non-reducing sugars, proves that under the influence of the equal temperature, average 9° C., they tend to reaching the same equilibrium concentration. This equilibrium concentration according to the principle of the moving equilibrium is clearly lower than the concentration attained at 4° C. on 9th October.

Both the lots too planted at 13° C., notwithstanding their differences at the time of planting, tend to attain the same equilibrium concentration which, as was to be expected, is lower than with the bulbs planted in the open.

Finally, the behaviour of LN 20—5/7 can still be mentioned. Its concentration is lower on 9th October than on 24th September, apparently because the concentration at 5° C. had then already become greater than corresponded to the equilibrium at 7° C.

The question arises whether the principle of the moving equilibrium can really explain everything here. The hydrolytical splitting of starch into sugars is a process which is attended with a very small evolution of heat and therefore only a slight equilibrium shifting can be expected.

BĚLEHRÁDEK (2) however points out that the position of an equilibrium is also shifted a.o. by viscosity- and chemical changes of the medium.

The possibility exists, that the bulb, at a lower temperature has a different percentage of water than at a higher temperature. Table II of the preceding publication (1) shows that LN 20—5/4—op indeed contains a little more water than e.g. LN 20—20—op. But LN 20—9—op contains as much as LN 20—20—op, although at 9° C. there are far more sugars. In spite of the slight differences in the entire amount of water, yet at a lower temperature a greater part of the water can be "free", be available to dissolve the sugars in it and in this way to dilute the concentration through which the sugar formation takes place more easily.

It is not inconceivable that the temperature influences one or more of the mentioned factors and in this way indirectly affects the rate of the sugar formation.

Although it has clearly appeared that before and shortly after the planting the non-reducing sugars are formed more rapidly at a lower than at a higher temperature, the behaviour of M 3 w 17—9—9/17 shows with

respect to M 3 w 17—9—9 that some time after the planting the sugar formation is on the other hand promoted by a higher temperature. Should this observation be correct, then this contrast is difficult to explain. Apparently during the course of the development an exchange of limiting factors takes place. At first the conversion is controlled by a process, which at a low temperature takes place quickest; after that by a process the rate of which increases by a rise of temperature.

Morphological development and carbohydrate metabolism.

Finally some remarks about the question which relationship there exists between the morphological development and the carbohydrate metabolism.

The quickest extension of the young plant is at about 13° C. in the period from 17th August until planting on 9th October. This low optimum can be explained by two factors:

1⁰. The direct temperature influence through which in the range of 5 to 20° C. the growth is quicker in proportion to the bulbs being in a warmer place.

2⁰. Through the greater store of non-reducing sugars at a lower temperature, the plant will have a tendency to grow more rapidly at a lower temperature. Presumably the reducing sugars here too are the direct source of food, but their forming is accelerated by a higher concentration of the non-reducing sugars.

Through the co-operation of both these factors, a curve is formed, the optimum of which is evidently at about 13° C.

In the introduction it has also been discussed, that the optimum shifts to higher temperatures in the course of the season. On the ground of the above, this is possible if the forming of the sugars is promoted by a higher temperature than is the case earlier in the season.

With the Murillo's of 1929—1930 the amount of reducing sugars during the first period after planting at 9° C. and in the open, is greater than at 17° C. (M 17—17—17). This may be partly due to the smaller concentration of the non-reducing sugars, and presumably partly to a stronger respiration at 17° C. After 1st January the reducing sugars of M 17—17—17 increase; even much faster than in the open with M 17—17—op. Round about this time there exists a possibility that the optimum is at a higher temperature.

The fact too, that previous cooling shifts the optimum quicker, can be connected with the carbohydrate metabolism. This already appears from the often discussed behaviour of M 3 w 17—9—9/17 with respect to M 17—17—17. The former lot can at 17° C. increase its percentage of reducing and non-reducing sugars quicker and is therefore already at an earlier date capable of growing faster at a higher temperature.

From this it appears that the "after-effect" of the cooling applied in

summer exists in an increase of the capacity so as to form a sufficient amount of sugars from the stored starch at a higher temperature.

If the lots of 1929—1930, which in the middle of December contain about equal amounts of reducing and non-reducing sugars were to be forced into bloom at 20° C. or higher in about that period, it would appear that the rate of extension of the plants is very different. Judging from their equal concentration of sugars this was not to be expected. The differences in the rate of stretching are however understandable, when we think, that as a result of their dissimilar summer treatment they have an unequally strong capacity to form useful sugars sufficiently fast at a high temperature.

This however does not mean that the "after effect" exclusively exists in the possibility of a more rapid sugar production. No more than the whole developing process can only be explained by the carbohydrate metabolism. We only intended to show that certain quantitative differences in the carbohydrate metabolism can be connected with quantitative differences in the rate of the morphological development.

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Laboratory for Bulbresearch.

Botany. — *Occurrence and transport of a substance causing flowering in the Soya bean (Glycine Max L.).* By J. KUIJPER and L. K. WIERSUM.
(Communicated by Prof. J. C. SCHOUTE.)

(Communicated at the meeting of October 31, 1936).

The question whether a substance causing flowering in plants exists, is still unanswered. In a recently issued publication KNODEL¹⁾ once more puts the question quite clearly; there are two principal views on the matter: one, indicated by KNODEL as the conception of KLEBS, to the effect that

¹⁾ H. KNODEL. Lässt sich die KLEBS'sche Ansicht über die Abhängigkeit der Blütenbildung von der chemischen Zusammensetzung der Pflanze aufrechterhalten? Z. f. Botanik. 29, 449 (1936).

flowering depends on a fixed ratio of soluble nitrogen to sugars in the plant, the other, first actually promulgated by SACHS, and assuming a specific substance causing flowering (Blütenbildender Stoff). KNODEL shows in his investigation that there is no proof of the existence of a fixed ratio between *N* and sugars which causes flowering; that flowering may at all events occur with very different values of this ratio and that in the same species, with a particular value of the ratio, flowering is sometimes seen to occur and sometimes not.

The question what causes flowering is once more brought into prominence by the investigations of what we will here call the Russian school; this school, whose spokesman is especially LYSENKO, but with whom various well-known physiologists, such as LJUBIMENKO, RASUMOV, MAXIMOV, BORODIN, DOLGUTSCHIN, and others cooperate, has drawn the special attention of botanists to the possibility of influencing the life-cycle of plants; they base their opinion on prior investigations of KLEBS, GASSNER, GARNER and ALLARD, etc. If it is possible strongly to influence the life-cycle, as is shown by the processes which LYSENKO summarizes under the name of "vernalization" to be actually the case, one is readily induced to think of specific substances, especially in the case of the so-called after-effect, which often brings about the transition from the vegetative phase to the reproductive phase weeks or months after the moment at which the factors which made this transition possible were present.

More especially the facts which are noticed in the photo-periodical reaction suggest the occurrence of such substances. The fact that a short-day plant reaches the reproductive phase when grown under a day of 12 hours, but does not develop flowers when grown under a normal-day (in our latitude), and that flowers are also produced when short day is given only during a short period after germination, in our opinion clearly points in that direction. In his previously cited investigations, short-day and long-day plants were the very ones with which KNODEL was unable to find any support for the view that it is a fixed ratio between *N* and carbohydrates that causes flowering. The very existence of an after-effect renders it in our opinion extremely unlikely that a particular ratio between assimilation products, formed long after the application of short day, should be responsible for flowering.

And finally the discovery of growth substances leads us instinctively to the conception of specific substances causing flowering.

Various investigators have already tried by means of direct experiments or by deductions from observations made for other purposes to answer the question whether substances promoting or substances inhibiting flowering occur.

Direct experiments have been made by KRASNOSELJSKAJA—MAXIMOVA ¹⁾,

¹⁾ T. A. KRASNOSELJSKAJA—MAXIMOVA, Russian, An attempt to elucidate the internal causes of retardation of earing in winterforms. *Trudy Prikl. Bot.* 27, 113 (1931).

who worked with winter-wheat. She tried to induce flowering (earring) by the application to the young plants of various sugars, asparagine and other N-containing compounds, yeast, etc., but without any result. No better result was obtained by injection into the young plant of an extract from flowering spring-wheat. When, however, an extract from germinating winter-wheat was injected into spring-wheat, a delay in earing was observed. From this observation KRASNOSELJSKAJA draws the conclusion that in germinating winter-wheat a substance is present which inhibits flowering.

SEREISKII and SLUCKAJA¹⁾ find no confirmation of this conclusion in their own experiments. They cut germinating seeds of winter-wheat transversely and fixed half seeds of vernalized and unvernallized wheat tightly together with plaster of Paris. No interaction between the vernalized and the unvernallized halves was observed, although the united plants grew well; no substance is translocated from one half to the other. Not being able to read the original Russian papers, we used summaries in English, so that we can form no judgment as to the details of this work.

GARNER and ALLARD²⁾ have made some observations in connection with the problem; soya beans, *which had already developed flowers* after short-day treatment, were subsequently cultivated under long-day conditions. These plants grow on vegetatively and do not develop any further flowers. It is possible to assume from this that there is no specific substance, but there is also the possibility that the production has ceased and that the substance formed has been used up during flowering. There is, however, no cogent reason to assume the presence of a specific substance; it is also possible to explain matters by assuming differences in the entire metabolic process in short-day and long-day plants. The authors themselves do not discuss this point.

Some Russian investigators made experiments on the question whether the effect of vernalization is reversible or irreversible; whether a fitness to flower, once induced, disappears, and vegetative growth sets in, when, after the induction, the specific factors for induction are no longer applied. LJUBIMENKO³⁾ cultivated soya beans under short-day conditions; when the plants flowered, they were transferred to long-day conditions. Shoots which developed under these long-day conditions did not flower; from this it is concluded that the main stem no longer has at this moment the capacity to produce reproductive organs, and that fitness to flower is reversible. If, however, a special substance be supposed to be the cause of flowering, it

¹⁾ A. SEREISKII and M. SLUCKAJA, Russian, On the nature of vernalization. Bot. Zurn. S.S.S.R.U. 19, 311 (1934).

²⁾ W. GARNER and H. ALLARD, Effect of the relative length of day and night and other factors of the environment on growth and reproduction in plants. Jrn. Agr. Res. 18, 553 (1919/20).

³⁾ V. N. LJUBIMENKO, Russian, On the theory of artificial regulation of the length of the vegetative period of higher plants. Sov. Bot. N^o. 6 (1933).

may be consumed by flowering in this case also, so that nothing is left to induce flowering in new shoots.

We may further mention experiments on localization of the photo-periodic effect. RASUMOV¹⁾ speaks in his publication of "transport"; this implies in our opinion a substance to be transported. He works with *Ullucus tuberosus* and *Solanum demissum*, plants which show a reaction to short-day treatment in forming tubers, and, as far as *Ullucus* is concerned, by changes in the top of the sprouts. RASUMOV darkens various percentages of the total leaf-surface, and states that only when a certain percentage is treated tuber-formation does start; the tuber-formation increases as a result of increasing the darkened portion above this limit. When only the middle part of a sprout is given short-day, the base and the top show a reaction; transport to the base is said to be better than that to the top. We think that no positive proof is given in these experiments of the presence of a transport substance.

Finally GARNER and ALLARD²⁾, working with *Cosmos sulphureus*, a short-day plant, darkened the top of the stem and treated the lower part with short-day. This part developed flowers, after which the top continued to grow, and the whole plant was exposed to normal day-length; the top then formed flowers also. The authors do not discuss this question, but we consider that the notion of a specific substance transported to the top of the plant suggests itself.

It will be clear from these facts that the problem whether specific substances are formed in vernalization or photo-periodic treatment is not yet solved; in many cases it was not even discussed; none the less we think that many indications pointing to its occurrence are present. A study of HITCHCOCK and ZIMMERMAN³⁾ gives in our opinion fresh support to this idea; they noticed that "synthetic substances" hastened flowering in a Turkish tobacco variety; this substance also inhibits growth, and it is worthy of note that in short-day plants short day causes not only flowering but also an inhibition of growth.

The first of the authors has worked for some years already on the vernalization of soya beans; as a result we have some varieties which have shown during 3 successive years that no flowers are produced under normal day-length when not sown before the end of April. They flowered in a 12—13 hour day. These species afford a good object for the study of the problem: does a substance causing fitness to flower exist? It should be present in plants treated with short-day, and absent in those cultivated

¹⁾ V. J. RASUMOV, Ueber die Lokalisierung der photoperiodischen Reizwirkung. *Planta* **23**, 384 (1935).

²⁾ W. GARNER and H. ALLARD, Localisation of the response in plants to relative length of day and night. *Jrn. Agric. Res.* **31**, 555 (1925).

³⁾ A. E. HITCHCOCK and P. W. ZIMMERMAN, Absorption and movement of synthetic growth substances from soil as indicated by responses of aerial parts. *Contr. Boyce Thompson Inst.* **7**, 447 (1935).

under normal-day. If such a substance should occur, its existence would be shown to be probable if one succeeded in transporting the fitness to flower by uniting a s.-d. soya (s.-d. = plant cultivated under short-day) with a l.-d. soya (l.-d. = plant cultivated under long-day) by grafting¹). In this way we tried to answer the question.

Methods.

We used two varieties of soya bean: Ked. 29, obtained from the Algemeen Proefstation voor den Landbouw at Buitenzorg, and soya à graines jaunes, obtained in 1933 from Vilmorin. This latter variety will be indicated below as the yellow soya bean.

We first tried whether it were possible at all to graft the soya bean, and succeeded in making whip-graftings of young soya bean plants. The plants were cultivated in the open air, and proved to be in the best condition for grafting during a period of about 10 days before and after the appearance of the first flowers, at the age of about 50 days. Grafting was performed by making an oblique cut at one of the internodes above the insertion of the cotyledons, sometimes at the first internode, but preferably at the 3rd to the 5th. The young stems should be rather rigid; for this reason the plants should not be cultivated in a hothouse.

For our grafting experiments we used a support with clamp in which the stem-top, i.e. the scion, was gripped, in order to be able to press it against the cut of the stock. Two thin pieces of raffia were then tied round the two parts and the whole was covered with grafting wax. It proved to be of the greatest importance to keep transpiration at a low level during the first few days after the operation; the plants had to be kept for at least 10 days under a glass bell in a cool place and in weak light. We put them for that purpose in a greenhouse. Reduction of transpiration could also have been attained by cutting the leaves, but we dared not do so, as the leaves might possibly have had something to do with the faculty of flowering.

After removal of the glass bell the plants stayed for another week in the greenhouse; it was then possible to transfer them into the open air. We were finally able on several occasions to achieve complete success, but for this the utmost care is necessary, especially during the after-treatment. We got the impression that it is favourable when the under-stem retains some leaves, that the grafting is therefore not performed directly above the cotyledons, but higher up the stem; moreover the axilar buds of the leaves sprout better than those of the cotyledons, axilar branches being necessary to control the influence of the scion on the stock.

For the following experiments the yellow soya bean was used.

All the seeds were sown on May 5th; germination followed on May 10th;

¹) We are glad to mention that it was Prof. Dr. W. H. ARISZ, of Groningen, who suggested the idea of grafting to us.

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SUBSTANCE CAUSING FLOWERING IN THE SOYABEAN (*GLYCINE MAX* L.).



Fig. 1. Grafted soya bean plants: left long-day stock, short-day scion; right short-day stock, long-day scion. X point of grafting; → axil with flowers photographed in fig. 2.



Fig. 2. Enlarged photo of axil with flowers on side-branch of long-day stock, represented in fig. 1.



Fig. 3. Soya bean plants 70 days after germination; right grown under short-day (9½ hour), left grown under normal-day.

the young plants were transplanted on May 14th. The s.-d. group was placed on May 18th on a flat truck, which was drawn every day at 7.30 a.m. out of a darkened shed and pushed back again at 5 p.m. Thus the plants were given a short-day of $9\frac{1}{2}$ hours; care was taken that the temperature did not rise too high in the shed. The l.-d. plants were given the normal-day at $53^{\circ}14'$ N.L.; on the day of sowing, May 5th, its length was already 15 hours.

On July 3rd 12 graftings were performed as follows:

6 s.-d. stocks were given a l.-d. scion,
 6 l.-d. " " " " s.-d. " "

After the operation all these plants were exposed to the length of the normal-day, although during the first period to weak light only (see above).

Besides the plants which were used for grafting, 2 plants, sown on the same day, were kept as controls; they were treated in exactly the same way as the experimental ones and so also placed under glass bells in the greenhouse; another group of 15 plants, sown on the same day, were kept during the whole summer in the open air.

Length of day.

Day-length in the latitude of Groningen ($53^{\circ}14'$ N.L.) is as follows:

April	1	approximately	13.00	hours
"	15	"	14.02	"
May	1	"	15.04	"
"	15	"	15.59	"
June	1	"	16.42	"
"	15	"	17.06	"
July	1	"	17.06	"
"	15	"	16.40	"
Aug.	1	"	16.00	"
"	15	"	14.50	"
Sept.	1	"	13.50	"
"	15	"	12.50	"

We noticed that yellow soya beans, sown on March 29th, still formed flowers. It is known that especially on young plants the influence of a short-day is a strong one; it may therefore be assumed that these plants showed a reaction after the day-length they were exposed to between April 1st and 15th, a duration, that is, of from 13 to 14 hours. As in previous years, plants sown at the end of April or the beginning of May never showed a reaction.

There is thus a possibility that plants which are grown after September 1st in normal day-length (13.50 hours) might show a short-day reaction. Although in other experiments we had noticed that in full-grown plants the reaction on short-day ($9\frac{1}{2}$ hours) only becomes visible after about one

month, we thought it better, in order to avoid false conclusions, to finish our experiments on September 15th.

Results.

The plants grown continually in short-day all formed flowers; general flowering started on July 8th. Plants grown under normal-day did not show any signs of flowering, just as we had noticed the last three years. It may be regarded as certain that the long-day plants used for these grafting experiments would have behaved in the same way and would have formed no flowers at all.

At the moment, thus, when the grafting was performed, (July 3rd), the s.-d. plants already showed many flower-buds. The various plants of each series showed very much the same habit, a fact which was always noted in soya bean plants grown under the same conditions.

Our 6 plants with s.-d. stock and l.-d. scion (top) behaved in the following way:

The leaves of the stock did not grow any more; often they were shed; new flowers did not develop; only in one plant a pod developed; side branches scarcely sprouted at all. The l.-d. scion, on the contrary, grew vigorously, and looked just like a normal l.-d. plant. On September 15th one of these plants showed a flower-bud on the scion.

The behaviour of the 6 plants with l.-d. stock and s.-d. scion was totally different. The scion hardly grew at all after grafting, and no more flowers were formed on it; the leaves became yellow and often dropped off. In one plant the scion at length died and was thrown off on August 1st. The l.-d. stock, on the contrary, formed vigorous side branches. The poor growth of the scion may, of course, be due to the sprouting and growth of these side branches, but we could not cut these, since, if flowering developed, it would necessarily occur on these branches.

The side branches have the vegetative type, as may be seen from fig. 1.

About August 27th 3 plants had the first flowers on the side branches; 2 showed flower-buds, and the plant from which the scion was thrown off on August 1st showed a flower on September 15th. (Fig. 2). All these plants thus reached the reproductive phase. The top end of the branches shows a slight inhibition of growth, the internodia are a little shorter, the development of hairs is somewhat less. We think that these features may be considered as signs of the reproductive phase; they form, however, only a weak indication in that direction.

As stated above, not one of the control plants showed any sign of flowering till September 15th.

Conclusions.

It seems to us that no other conclusion can be drawn from these facts than that fitness to flower has a material carrier, which is transported

from the s.-d. plants over the grafting point to the l.-d. plants. In one series the transport took place in the basal direction, in the other in the apical direction, but this happened only in one plant, so that there is no absolute certainty in this case. Where the direction was basal flowering started earlier and more generally (in all plants); basal transport seems to occur more easily.

Whether the material carrier should be considered as a specific flowering hormone, or another type of planthormone, or a specific carbohydrate or nitrogenous compound formed in the assimilation process, cannot be concluded from these experiments. It seems clear to us, however, that no fixed ratio N: carbohydrate can be considered as the cause of flowering, for the side branches and the scions have developed under exactly the same conditions as the l.-d. control plants; so that the same ratio did not have any such effect in these plants. It further follows from these facts that transport seems to take place better in the basal direction than in the other direction, which is in agreement with RASUMOV's observation (see above). That transport actually does take place looks still more probable owing to the fact that flowering starts first in the first side-branch under the grafting point, at a point, that is, nearest to the place of origin, namely, the scion. We would also draw attention to the fact that photoperiodicity in soya causes flowering in combination with inhibition of growth. GARNER and ALLARD (1919/20) have already observed this, and in this variety also this phenomenon is pronounced, as may be seen in fig 3. We therefore think that our observation of a slight inhibition of growth in the side branches is not without importance; it would seem to prove that both fitness to flower and inhibition of growth are, in this case also, transported from scion to stock or vice versa.

There is a certain discrepancy between our observations and those of LYSENKO (cited in *Vernalization and Phasic Development of Plants*, Imperial Bureau of Genetics, 1935). He made cuttings from flowering soya plants, which he cut above and below the place where flowers were inserted. Those taken from above this place produced flowering plants, the other ones did not flower. In cotton plants, after flowering, side branches sprouted; those formed above the flowers yielded flowering sprouts, the lower ones did not. This seems not to agree with our observation of easier basal transport.

After the series of graftings about which we have reported above, other series were made in the latter part of July and during the month of August. In the middle of September no flowering was observed in these plants; for the reasons stated above we then had to finish this experiment, as results, however they might turn out, could not have been entirely reliable. This was regrettable, for in these series we also had control plants which differed from the grafted ones in one respect only, viz., a l.-d. scion was grafted on a l.-d. stock, in order to see whether the operation of grafting

in itself would have any influence. It is, however, very unlikely that there would be any influence, since GARNER and ALLARD and other investigators have already shown that no other environmental condition but the relative length of day and night has any effect at all.

Summary.

It is shown that in the soya bean (*Glycine Max* L) when a flowering scion is grafted on a stock treated with long-day, that is, without fitness to flower, flowers develop on side branches of the stock; conversely a short-day stock causes flowering in a long-day scion. A substance causing this phenomenon thus passes the point of grafting; transport in the basal direction seems to be much easier than in the apical direction, for flowers were formed on the long-day stock earlier and more profusely than in the converse case.

Groningen, September 1936.

Geology.—*On the Geology of North Banka (Djeboes).* By J. WESTERVELD.
(Communicated by Prof. H. A. BROUWER.)

(Communicated at the meeting of October 31, 1936).

The author surveyed the district Djeboes¹⁾ (N. Banka) in August—September 1933, and the following lines contain the results of his preliminary exploration.

The rock-formations of Djeboes are very few and consist of a monotonous series of presumably triassic slates and sandstones, with a large intrusive granite-mass and locally sills of uralitic diabase, the latter being older than the granite. The granite forms a country of gently sloping hills, amongst which the highest tops only reach between 150 m. and 200 m. above sea-level (G. Klabat, G. Ganten). The sediments for a large part show the same little pronounced topography as the granite. Only where hard banks of sandstones predominate over slates the sedimentary formation forms rather high and steep ridges (G. Mempari = 205 m.); in the district Soengeilat S. E. of Djeboes the highest elevations of Banka are even found on outcrops of sediments (G. Maras, \pm 700 m.). On the Penjaboeng-promontory W. of the granite-country hard sills of

¹⁾ All Malayan names are written according to the Dutch orthography of Malayan words; oe = pronounced as u. Tg = tandjoeng = cape. G = goenoeng = mountain, P = poelau = island. S = soengai = river. A = aer = water or rivulet. K = kampoeng = village. Bt = boekit = hill.

diabase constitute an isolated small chain of hills with steep flanks, separated from the granite-hills by marshy ground.

THE GRANITE. The granite of Djeboes forms part of the larger long granite-mass, which stretches over a distance of nearly 100 km. through the districts Djeboes, Blinjoe and Soengeiliat in N. Banka, and occupies the whole northern section of the investigated region. Solid rock-exposures are to be seen along the north-coast between Tg. Lesoem and Tg. Melalo, along the Klabat-Bay, and on small islands along the coast (P. Tenoeng, P. Pemoedjah, P. Kambing). Farther inland the granite is deeply weathered and does not show any solid outcrops; some of the hills and watersheds however are covered with rounded boulders (G. Ganten, G. Klabat, and others), remnants of disintegrated monadnocks. The process of disintegration can be studied in all its phases along the coast from splitting along master-joints up to the development of the typical woolsack-forms.

The northern boundary of the granite is covered by the sea and lies beyond the granite-cliffs Malang Dojong and Malang Hijoe. The southern boundary against the sediments is nowhere exposed at the surface and has been deduced from drillhole-reports and extrapolation of scattered observations.

Macroscopically the granite is of very uniform character over the whole area and of coarse-grained porphyritic structure. The phenocrysts are all Karlsbad-twins of orthoclase of an average length of 5—6 cm, sometimes 8 cm. A finer grain is sometimes met with near the contacts with the older rocks (shales and diabase), e.g. on P. Kambing in Klabat-Bay and above the beach S. E. of Tg. Lesoem. A parallel orientation of the feldspars has locally been observed, but on the whole is not very obvious.

Microscopically it is a biotite-granite, in most cases consisting of quartz, orthoclase, oligoclase, biotite and subordinate amounts of zircon, apatite and orthite. Quartz shows xenomorphic development and is the last-formed mineral. It often contains inclosed biotite, plagioclase, zircon and apatite, and sometimes shows graphic intergrowths with orthoclase. The extinction is mostly undulous. Orthoclase, apart from its occurrence as large phenocrysts, has irregular boundaries in the coarse groundmass, in which it is also developed as Karlsbad-twins. Plagioclase (oligoclase) is often inclosed by the alkali-feldspar, which in addition regularly contains strings of albite (micropertite). Microclinisation appears as a result of stress in some instances. The oligoclase often shows a zonal arrangement of rims with different extinction-angles, while seams of myrmekite against orthoclase are of common occurrence. Apatite and zircon are common and sometimes even rather abundant constituents; zircon in short prisms with well-developed terminal faces. These minerals are frequently inclosed by biotite, in which case the sometimes coarsely crystallized zircons are surrounded by dark pleochroitic halos. The biotite is locally altered into chlorite, iron-ore and sphene, the latter mineral proving a certain

Ti-content of the dark mica. Mica from a specimen of dike-granite from the Toboali-district (S. Banka) reacted upon tests for Li and F²⁾, which probably are also present in granite-biotites from other parts of Banka. Green amphibole has only been found in a granite-specimen from P. Tenoeg. *Muscovite* (sericite) and *epidote* are alteration-products of the feldspars. *Orthite* seems to be regularly distributed in the granite-mass. It could be detected in 13 out of 33 microscopic slides from granitic rocks. The mineral occurs as slender prisms, elongated along the b-axis, in rather coarse development. It is bright yellowish-brown, often with a brownish-green core, and has its birefringence-colours masked by the strong own colour. The mineral is often accompanied by or inclosed in biotite and in the latter case it is surrounded by a dark pleochroitic halo. The presence of the rare earths as a constituent of the granite is rendered conspicuous by the common occurrence of this Ce-epidote.

The rare earths evidently play an important part in the geochemistry of the inner Malayan Arc. Monazite is a common companion of cassiterite in the streamtin-beds of Malaya³⁾ and the Tin-islands. VAN TONGEREN recently found more than 1 % La_2O_3 and 5—10 % Ce_2O_3 in a cassiterite-concentrate from Banka⁴⁾ and monazite has been detected microscopically by DRUIF in granites, biotite-gneisses and biotite-schists from Poeloe Berhala in the Strait of Malacca⁵⁾, while ESENWEIN⁶⁾ determined orthite in aplite-granites from the island Karimata between Billiton and Borneo.

The granite along the coast sometimes contains dark streaks of biotite alternating with leucocratic bands, e.g. W. of K. Penganak. Large orthoclase-crystals are developed independently with regard to the streaks and therefore must be of earlier crystallization.

Xenoliths of intrusive rocks of acid to intermediate character, but always richer in dark constituents, are rather common in the granite. Petrographically their composition is granitic to dioritic. Most inclusions are of fine-grained biotite-rich granite with the same constituents as the surrounding rock, but they often also contain green amphibole. The more melanocratic xenoliths are sometimes nearly devoid of orthoclase and quartz and can be called mica-diorite. Orthite is also a common mineral in these inclosed rocks. The xenoliths evidently are remains of the first

²⁾ R. D. M. VERBEEK, Geologische Beschrijving van Banka en Billiton. Jaarboek v. h. Mijnwezen in N.O.I. (1897). With atlas.

³⁾ J. B. SCRIVENOR, The Geology of Malayan Ore-Deposits, p. 143—145.

⁴⁾ W. VAN TONGEREN, Mineralogical and chemical Composition of the Syenite-granite from Boekit Batoe near Palembang, Sumatra, Netherlands East Indies. Proc. Roy. Acad., Amsterdam, **39** (1936), p. 670.

⁵⁾ J. H. DRUIF, Over gesteenten van Poeloe Berhala (Straat van Malakka, Gouv. S.O.K.). Proc. Roy. Acad., Amsterdam, **38**, p. 639—649 (1935).

⁶⁾ P. ESENWEIN, Die Eruptiv-, Sediment- und Kontaktgesteine der Karimata-Inseln. Wetensch. Mededeelingen No. 24, Dienst v. d. Mijnbouw in Nederlandsch-Indië, p. 8 (1933).

chilled roof of the large batholith and were stopped down by the intruding granite-magma. Their former marginal position is revealed by the finer grain and the abundance of dark minerals. The roof of the batholith evidently cannot have been far above the present topography.

Black tourmaline (schorl) and *fluorspar* are sometimes very abundant in the granite. They are later than the above-mentioned granite-minerals. "Suns" and nests of tourmaline are very conspicuous at various spots along the coast, e.g. south of Tg. Pemoedjah. Farther inland tourmalinization is widespread in weathered granite-floors of abandoned eluvial tin-workings, as around K. Ketap, and it was observed by the author in the granite-bedrock of a streamtin-mine along the Boenoet valley, not far from the granite-boundary.

Aplite-veins do not seem to be very common in the granite. They were observed locally on P. Tenoeng and in granite-exposures along the sea-shore W. of K. Penganak. Their width is very moderate (1—2 dm.) and mineralogically they consist of quartz and feldspar with nearly no biotite. Pockets of *pegmatite* locally accompany the aplite-veins. Fine exposures of this type are found S. of Tg. Pemoedjah, where streaks of pegmatite in the granite of a width of ± 20 cm. are bordered on one side by aplite. The pegmatites contain the same minerals as the granite, and in addition much black tourmaline and some later quartz. Under the microscope the tourmaline shows blue rims around a brownish-violet core. It replaces orthoclase along cleavage-planes. The association of aplite and pegmatite of normal granite-composition in the same fissure is of common occurrence. It is explained by NIGGLI⁷⁾ by a variability in physical circumstances governing the intrusion of residual liquors of granitic magmas, e.g. rate of cooling and opportunity given to the volatile substances to escape. Slow cooling and retention of volatiles promotes the formation of pegmatitic material; in the reverse case aplites will form.

The boundaries between tourmaline-bearing pegmatite-strings and granite are often less sharp. The volatile constituents apparently gathered in the fluids and crystallized with the residual silicate-material not far from the spots, where they were concentrated. Their congealing seems to have taken place, when the granite still was in a semi-viscous state. The volatile elements, like boron and water-vapour, attacked the already formed orthoclase, so that schorl was formed before the complete solidification of the granite and its inclosed pegmatite-streaks.

Tourmaline-veins, following sets of rather strictly parallel and steeply dipping master-joints and directed according to the longitudinal axis of the batholith, are very numerous in the eastern part of the granite-region. They are abundant along the promontory W. of Tg. Melalo and along Klabat-Bay from Tg. Melalo until P. Kambing. The *black tourmaline* of these veins is accompanied by *muscovite*, *sphene*, epigenetic quartz,

⁷⁾ GRUBENMANN—NIGGLI, Die Gesteinsmetamorphose I, p. 325—326 (1923).

arsenopyrite and *fluorspar*. The tourmaline of these veins is clearly later than the schorl of the pegmatite-pockets and the tourmaline-suns in granite. Its volatile constituents and those of the accompanying minerals have escaped along joints in the already solidified granite-crust after their concentration at greater depth. Cassiterite has not yet been detected microscopically in the pegmatite-pockets, neither in the tourmaline-veins. The tin-oxide however certainly must be a minor associate of the pegmatitic-pneumatolitic schorl. Arguments on this question will be given in a next paper on the kinds of mineralization along the Malayan Tin-Belt.

The intrusive character of the granite with regard to the sediments can be deduced indirectly from the following observations: *a.* The existence of chilled margins of finer grain; *b.* the occurrence of hills of limonitized shales and quartzified sandstones along the granite-boundary, where limonite has been formed by the oxidation of pyrites; *c.* the occurrence of diabase-inclusions from the diabase-sills of the Penjaboeng-promontory in fine-grained granite S.E. of Tg. Lesoem; *d.* the occurrence of arsenopyrite-quartz veinlets and pyrometasmatic diopside in slates between the diabase-sills of G. Penjaboeng near the granite-boundary, etc. The intrusive nature of the Banka-granites has already been demonstrated conclusively by VERBEEK ⁸⁾ in the Toboali-district and is confirmed by all later observations, e.g. for the granites of Muntok ⁹⁾.

THE DIABASE. The Penjaboeng-promontory W. of the granite-area consists of a number of sills of diabase, intruded more or less conformably between intensively disturbed and contorted shales and sandstones of the mesozoic sedimentary formation. Their dip is between 40° and 60° N.N.W. and their thickness varies between \pm 60 and 500 meters. The hardness of the rock makes them rise as a crest of steep hills above the adjacent low granite-country. Tg. Lesoem and the smaller projecting capes W. of G. Penjaboeng owe their existence to the difference in resistance between the dark intrusives and the intercalated strips of sediments. VERBEEK ¹⁰⁾ considered the rocks from Penjaboeng as a series of "diabase"-flows, alternating with siliceous shales, suggested to be silicified "diabase"-tuffs of the same geological age as the "flows". The author's observations however do not agree with this view. The dark doleritic rock is clearly intrusive into the folded sediments, which do not show a trace of volcanic material and on macroscopic and microscopic examination appear to be of quite the same character as the ordinary sedimentary formation along the southern margin of the granite. The diabase does not show any cataclastic structure. It should be considered as a fore-runner of the granite and therefore cannot be compared with the permo-carboniferous "Pahang

⁸⁾ R. D. M. VERBEEK, l.c., p. 95.

⁹⁾ Quarterly Report of the Mining Department in the "Javasche Courant", Jan. 12th 1934, No. 4, p. 6. (Dutch).

¹⁰⁾ R. D. M. VERBEEK, l.c., p. 31.

Volcanic Series" of the Federated Malay States as has been done lately¹¹⁾. Apart from its independence of folding, which only affected the sediments, the intrusive character of the diabase is also shown by the strong silicification of the shales and sandstones along the margins of the sills.

Macroscopically the rock is of a dark green colour and a fine grain. On exposures W. of G. Penjaboeng it is pronouncedly porphyritic with phenocrysts of altered plagioclase of 1 cm. and more.

Microscopically the rock shows a typical ophitic texture. Its primary constituents are *plagioclase* and *pyroxene* with some accessory *ilmenite*. By hydrothermal action a series of secondary minerals have developed out of the original constituents, mainly *uralitic amphibole*, but also *prehnite*, *epidote* and *pyrite*. The plagioclase (basic andesine) is lath-shaped and shows some alteration into secondary uralite. Nearly colourless pyroxene has been observed as scattered remains in a single specimen. The mineral fills up the spaces between the plagioclase-crystals, but mostly has been entirely metamorphosed into aggregates of uralite-needles, which show a pleochroism between light-green and brownish-green.

Not far from the spot, where the contact between the dark intrusive and the younger granite plunges below the diabase and its intercalated strips of sediments, the granite contains inclusions of diabase, stoped from the older intrusive during the upward movement of the later acid magma. The exposure can be seen above high water-level at the base of the steep diabase-hill. The diabase itself, a little farther towards the N.W., was seen cut by a quartz-veinlet. On microscopic examination this veinlet appeared to consist of quartz with coarse fibres of uralite, sphene and prehnite, while the immediately adjacent diabase showed itself to be nearly entirely converted into an aggregate of uralitic amphibole, chlorite and prehnite. The intrusive nature of the granite with regard to the diabase is therefore clearly proved and the uralitization of the diabase undoubtedly has been caused by water-vapours emanating from the underlying granite-magma. Uralitization and prehnitization proceeded from clefts in the melanocratic rocks, which were followed by the volatile agents during their hydrolysing action.

THE SEDIMENTARY SERIES. The sedimentary series of Djeboes occupies the whole area south of the granite-country and is intercalated as narrow strips between the diabase-sills of the Penjaboeng-promontory. It belongs to the main sedimentary formation of Banka, which occupies practically the whole island outside the granites. The best exposures are along the sea-shore between the mouth of S. Boeloeh and Tg. Ganting (foto 1), and N.E. of this cape, where they are laid bare at low tide. Farther inland the series is only directly observable on the flanks of some steep hills (G. Mempari, Bt. Kemoening). Occasionally its layers can be

¹¹⁾ J. ZWIERZYCKI, Enkele nieuwere geologische waarnemingen op de tineilanden en op Sumatra betreffende het tinvraagstuk. "De Mijnningenieur", p. 172 (1933).

seen in the bedrock of alluvial tin-mines. The formation consists of a monotonous alternation of dark shales and yellowish sandstones. The latter are often developed as powerful banks, which project by their greater hardness. No stratigraphical division has as yet been made because of the lack of fossils and guide-horizons, and the complicated folding of the series. The relative rarity of exposures also hampers its detailed investigation.

Microscopically the sandstones show an aggregate of more or less rounded quartz-grains with undulous extinction. The grains are cemented by a matrix of amorphous clay-material with small flakes of sericite. The clay-substance sometimes predominates, so that we get transitions between shales and sandstones. The undulous extinction of the quartz-grains proves their provenance from an older land-mass. As such a schist-formation, of which the existence over limited areas in the districts Blinjoe and Soengeiliat has lately been observed by J. ZWIERZYCKI in the bedrock of S. Boeboes and in the mine Bt. Pemali¹²⁾, may be regarded.

A triassic age has been accepted for the sedimentary series of Banka on account of their close similarity to the triassic series of quartzites, phyllites and shales, which occupies large areas E. and W. of the Main Range in the Federated Malay States, where its age has been proved by the occurrence of lamellibranchiate faunae¹³⁾. The discovery of casts of *Daonella* sp. in brownish-gray shales near Tg. Limboeng on the island Lingga of the Riouw-Lingga archipelago is of importance in this connection¹⁴⁾. A further support to the supposed triassic age is given by the discovery of remains of radiolaria in more or less silicified shales from the Penjaboeng-promontory (plate 2). I had the opportunity to compare the specimens with a sample of radiolarian shale from Kedah, presented to the Geological Laboratory at Bandoeng (Java) by Mr. J. B. SCRIVENOR. In addition a specimen of radiolarian shale was available from the underground-works of the Klappa Kampit tin-mine on Billiton, from which island the existence of radiolarian chert has been known since VERBEEK's journey in 1894—1895¹⁵⁾. The rocks from all three localities showed quite the same characteristics, both macroscopically and microscopically and therefore may be approximately of the same age. SCRIVENOR¹⁶⁾ describes the bulk of the radiolaria-cherts from Malaya as lying at the base of the triassic sandstones and shales and immediately above

¹²⁾ J. ZWIERZYCKI, l.c. The VERBEEK-collection in the Geological Laboratory at Bandoeng (Java) contains some specimens of true schists from Blinjoe, which however have not been recognised as a separate and older formation by this author.

¹³⁾ J. B. SCRIVENOR, *The Geology of Malaya*, p. 62—71 (1931).

¹⁴⁾ A. CHR. D. BOTHE, *Geologische verkenningen in den Riouw-Lingga Archipel en de eilandengroep der Poelau Toedjoeh (Anambas- en Natoena-eilanden)*. *Jaarboek van het Mijnwezen in N.O.I.* (1925), *Verhandelingen*, Vol. 2, p. 143.

¹⁵⁾ G. J. HINDE, Note on a Radiolarian Chert from the island of Billiton. In: VERBEEK, l.c., p. 223.

¹⁶⁾ J. B. SCRIVENOR, *The Geology of Malaya*, p. 76.



Foto 1. Alternating bands of sandstone and shale (triassic) on the beach between the mouth of S. Boeloeh and Tg. Ganting; exposed at low tide.



Foto 2. Cliff of diabase at Tg. Lesoem, divided into angulous blocks by numerous partings. The steep N.E. master-joint system is most clearly developed.

carboniferous limestone, but mentions the probability of their occurrence higher in the triassic series.

The coloured geological map of Banka, scale 1 : 100.000, edited in 1920 by the Government Tin-Mines, distinguishes a narrow zone of mica-, chlorite-, and talc-schists S. and S.W. of the bow-shaped, long granite-mass of N. Banka, on both sides of Klabat-Bay. As far as we know no definite proof has however been given of the existence of rocks older than the triassic series in Djeboes. The spots on the old map, where the schist-formation is supposed to outcrop, are occupied by granite or dark shale. The only occurrences of really unmistakable schists are in Blinjoe and Soengeiliat, as referred to before. The direction of trend of the sediments is nearly always N.W.-S.E. or W.N.W.-E.S.E. and therefore roughly follows the boundary of the granite-intrusion. The dips are unvariably steep, often vertical, as near Tg. Ganting. On the Penjaboeng-promontory the strike of the strongly crumpled and contorted sedimentary strips between the diabase-sills is about N.E., which may be regarded as an indication, that the nose of an anticline plunges below sea-level W. of the mouth of S. Bembang. VERBEEK, on his geological map of Banka, indicates a N.E.-S.W. to N.N.E.-S.S.W. trend of the sedimentary series on the islands P. Nanas and P. Beriberi in Klabat-Bay. This supposed sudden change of direction induced him to draw a fault-line along Klabat-Bay and to explain the origin of this wide and extended inland-sea by the easiness the brecciated rock-formations along the fault-plane yielded to corrosion¹⁷⁾. On the authority of VERBEEK the Klabat-Bay fault has been accepted by later authors¹⁸⁾. In order to verify this question these islands were visited again. P. Medang, P. Langkoeas and P. Beriberi appeared to be hard outcrops of intensively limonitized shales without any trace of bedding. The P. Nanas-group consists of hard silicified sandstones, cut by innumerable quartz-veinlets and neither shows any trace of bedding-planes. The supposed bend in the trend of the sedimentary series therefore must be denied. It is besides neither confirmed by the course of the boundary-line between granite and sediments, which can be followed across Klabat-Bay without any break. The inland-sea simply represents a drowned main valley in the mountain-ranges of the former Sunda-land; S. Antan, S. Lajang and S. Simboeboer have been its former head-waters.

The apparent enormous thickness of the sedimentary series — VERBEEK accepted some 20.000 meters — undoubtedly must be ascribed to intensive folding, ev. to overfolding and imbricated structures, which caused a repetition of series. The narrow zones of metamorphic schists

¹⁷⁾ R. D. M. VERBEEK, l.c., p. 37.

¹⁸⁾ Compare for instance the geotectonical map of the Netherlands East Indies by J. ZWIERZYCKI in "Jaarboek v. h. Mijnwezen in N.O.I.", Verhandelingen (1929), Plate Xa, scale 1.500.000.

in the districts Blinjoe and Soengeiliat can best be explained by reversed faulting, although we have no direct proof.

Along the granite-boundary the series has undergone the influence of hydrothermal fluids from the deeper granite-magma. In fresh exposures the black shales near the granite — as was seen by the author in the bedrock of an alluvial tin-mine in the valley of S. Ongkai — are impregnated with pyrite. By oxidation near the surface the pyrites leave concretions, veinlets and boulders of ironstone, limonitized shales, etc. This feature has long since drawn the attention of mining-officials and can be studied in many hills, e.g. S. of G. Mempari (Bt. Kendi, Bt. Beloe-kang), along the highroad between K. Kapit and K. Telak, and on some of the islands in Klabat-Bay already referred to. The sandstones sometimes show strong silicification and veinlets of secondary quartz, as on Bt. Menamar and on the western island of the P. Nanas-group.

On the Penjaboeng-promontory near Tg. Lesoem the radiolaria-bearing siliceous shales are cut by veinlets of quartz with some arsenopyrite and chalcopyrite, while some slides show streaks of hydrothermal quartz and colourless pyroxene, originated by the pyrometasomatic action of the underlying intrusive granite. The quartz-veins contain a slight amount of Ag (15 gr./ton) and much less Au and Cu, but no Sn. Some grains of gold from tributaries of S. Antan, to be seen in the Banka-collection at the Geological Laboratory at Bandoeng (Java), may come from similar quartz-veins. Quartz-veinlets with cassiterite have formerly been found between shales directly N. of K. Poepoetbawah and also S.W. of this village¹⁹).

STRUCTURE OF THE GRANITE. All along the north-coast the solid granite-exposures show a pronounced development of steeply dipping master-joints, accompanied by less obvious cleavage-systems. The most important of joint-systems is that following the direction of the longitudinal axis of the batholith. It runs slightly S. of E., rarely N. of E. In the western section of the granite-area the development of this system can be studied on rock-faces near the mouth of S. Bembang, S. E. of Tg. Lesoem, on the island P. Tenoeng, S. of Tg. Pemoedjah, and on G. Ganten. It is also slightly developed in the diabase-cliffs of Tg. Lesoem, with a turn towards the S. E. Between Tg. Pemoedjah and the mouth of S. Jeboe on the north-coast other systems prevail, but from Tg. Sangau to Tg. Melalo and along Klabat-Bay as far as P. Kambing the "longitudinal" system is again the most important. In this eastern section of Djeboes the main fracture-system in the granite is besides rendered very conspicuous by the development of thin tourmaline-veins on the joints. Their mineralogical composition has been described on a preceding page. These tourmaline-veins were also seen on G. Ganten and in the weathered floor of abandoned eluvial tin-mines near K. Ketap. Towards the Penjaboeng-promontory

¹⁹) R. EVERWIJN, In "Jaarboek v. h. Mijnwezen in N.O.I.", 1, 151 (1873).



these tourmaline-veinlets seem to be less numerous (plate 1). Diagonal and transverse cleavage-systems are well developed in the western half of the granite, although they are by no means absent along Klabat-Bay and W. of Tg. Melalo. Especially interesting are the diagonal N.E. joints on the Penjaboeng-promontory and its immediate neighbourhood (plate 2). The diabase-cliffs are sharply cut into angulous fragments by the N.E.-system (foto 2), which has also been observed in sediments E. of Tg. Lesoem, where it has partly been occupied by sulphide-quartz vein-stuff. The system can also be followed on the first granite-exposures S. E. of Tg. Lesoem, on P. Tenoeng, and faintly near the mouth of S. Bembang. An aplite-veinlet on P. Tenoeng follows the N.E. trend. The continuation of these diagonal fractures from the granite into the diabase across the intrusive contact between these rocks reveals that they owe their existence to the development of master-joints in the granite after the chilling of its roof. The granite plunges away under the diabase-sills and its cleavage-structure was transplanted on the overlying older rocks. The continuation of cleavages into the intruded rocks along the margin of granite-masses is of widespread occurrence and has been described clearly by CLOOS for the granite of the Riesengebirge in Silesia²⁰). Which of the joint-systems is oldest cannot be indicated with certainty. For the eastern section of the granite-area it seems most likely that the E.-W. system, occupied by tourmaline-veinlets, is the oldest, because the volatile constituents certainly must have escaped from their deeper locus of concentration upwards through the chilled cupola as soon as the first contraction-cracks formed. The N.-E. system on and near the Penjaboeng-promontory too must belong to the earlier partings, because its trend is locally followed by an occasional aplite-dikelet and by quartz-veins of the hypothermal type. The N.-S. joints of Tg. Melalo and S. of Tg. Pemoedjah are evidently later. They are not mineralized and were seen to brecciate pegmatite-aplite veins on the latter locality.

"Longitudinal" and cross-joints are also locally developed in sediments between Tg. Ganting and S. Boeloeh. Whether they correspond to cleavage-systems in the granite could not be ascertained by lack of exposures.

Pseudo-bedding of the granite by alternating streaks of leucocratic and melanocratic minerals seems to be rare. Some parallel arrangement of orthoclase-phenocrysts has been observed locally.

The "longitudinal" cracks dip vertically in the southern portion of the granite-mass, e.g. between P. Kambing and the coast-section N. of G. Klabat. Near Tg. Melalo however the dip is steeply S., so that the arrangement of the tourmaline-veins is asymmetrically fan-shaped.

The origin of these main cleavages probably has been initiated by contraction-clefts in the chilled roof of the cooling batholith, eventually also

²⁰) HANS CLOOS, Tektonische Behandlung magmatischer Erscheinungen. I, Das Riesengebirge in Schlesien (1925).

by some directed fabric of its mineral constituents, inherited from the period of upward flow in the still partly liquid granite-magma. They were however accentuated by the vertical push exerted by the still uncrystallized magma-portion below²¹). No dragging seems to have followed upon the parting: the minerals from the tourmaline-veins shortly after the opening of the joints sealed the walls together again and do not show brecciation. These longitudinal master-joints apparently are the fissures, along which "pneumatolytic" and "hydrothermal" agents could escape into the roof of the batholith and into the older sedimentary series, where they deposited their content of Sn, Fe, B, S, As, F, etc.

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²¹) Compare the arguments brought forward by H. CLOOS in: Einführung in die Geologie, p. 82—101 (1936).

Palaeontology. — *On some Caprinids and a Monopleurid from Southern Santa Clara, Cuba.* By A. A. THIADENS. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of October 31, 1936).

In 1933 several specimens of Caprinidae and Monopleuridae were collected in Cuba by the Utrecht geological expedition. The localities can be found on the sketchmap (f. 1). The material is in the Geol. Institute of the Utrecht University. I am much indebted to Mr. H. J. MAC GILLAVRY whose extensive knowledge about Rudistids always was kindly placed at my disposal.

All different species have been found in the same formation. It is rather difficult to state its age. We found a fauna containing species from the genera *Coalcomana*, *Caprinuloidea*, *Sabinia*, and ?*Tepeyacia*. A comparable fauna is known from Mexico. There, according to G. BOEHM (1) *Caprina ramosa* [this is *Coalcomana ramosa* (G. BOEHM)] occurs associated with several other fossils in the "Escamela Kalk" of East Mexico. BOEHM thinks we would not be far from the truth if considering the "Escamela Kalk" to be young cenomanian. DOUVILLÉ (3), although regarding the cenomanian age of these deposits to be probable, points to the possibility, that in the "Calcaires d'Escamela" several horizons of rather widely different age (Aptian to Cenomanian) are represented. PALMER (7), who found *C. ramosa* together with a.o. *Caprinuloidea* in Soyatlan de Adentro, Jalisco, Mexico, refers it to the Cenomanian. C. BURCKHARDT (2) too places the "Escamela Kalk" into the Cenomanian, but he mentions the possibility of its containing equivalents of the upper Cretaceous. I think we safely can adopt the cenomanian age of *C. ramosa*, although it may be younger.

Caprinuloidea, found together with *Coalcomana ramosa* in a fauna which finds its "closest counterpart in the Schiosia beds near Termini-Imerese in Sicily" [very probably of cenomanian age according to DOUVILLÉ (3)] has been referred by PALMER to the Cenomanian. *Sabinia* occurs in Europe and Bithynia in the Senonian. PALMER found several new species of *Sabinia* in Paso del Rio, Colima, Mexico, together with *Immanitas* n.g., *Palus* n.g., *Apricardia asymmetrica* n.sp., *Horiopleura gregaria* n.sp. and refers it to the Cenomanian. *Tepeyacia* has been found by PALMER in Tepeyac, Puebla, Mex., in a "very pure limestone". As PALMER does not mention other fossils associated with *Tepeyacia*, it seems difficult to establish the exact age. PALMER places the Tepeyac limestone in the Turonian for its close resemblance with the Huescalapa limestone (Jalisco) containing several *Radiolites*, *Sphaerulites*, *Requienia* and *Bayleioidea*. However, these being all new species the turonian age seems uncertain.

Apparently in Mexico the age of *Sabinia* and *Tepeyacia* is uncertain, that of *Coalcomana* and *Caprinuloidea* very probably cenomanian or younger. In Cuba we found *Caprinuloidea*, *Tepeyacia* and *Sabinia* in the same locality (L328). We are inclined to refer our cuban fauna to the cenomanian with the possibility of turonian age.

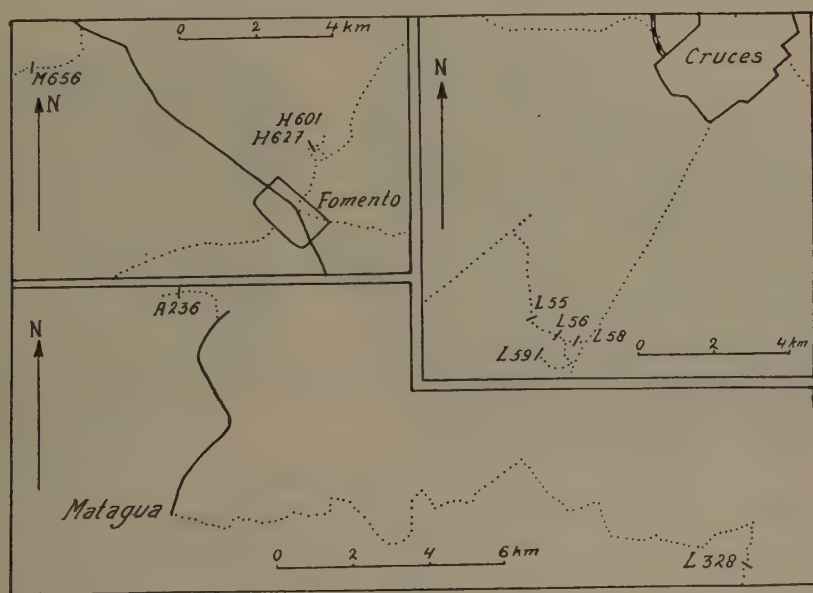


Fig. 1.

?*Tepeyacia corrugata* PALMER [f. 2, 5 (9, 10)]. Occ. Papers Cal. Acad. Sc. XIV. 1928, pp. 46—47, pl. VII. f. 4—5.

At L328 have been found fragments of several specimens in solid rock. Only sections and weathered-out surfaces have been studied. The shell is conical, spreading quickly. In transverse section the diameters are 28×17 , 32×24 , 45 and 33 mm. There are about 20—30 sharp ribs. The shell is built

up by two layers. The outer one consists of funnel plates only, thus having a compact lamellar structure. In the *Radiolitinae* this structure, if present, apparently occurs only in the so-called "structure mixte" (6, p. 236). The funnel plates are strongly undulated, the downfolds corresponding to the ribs. The badly preserved inner layer of the shell does not show a special structure. Very conspicuous is the presence of two more or less deep folds of the shell, marked on the surface by two channel-like areas. In *Tepeyacea corrugata*, PALMER considers these to be the two siphonal zones S and E, with which we agree. At the side of these zones the ribs on the surface are irregularly arranged. The opposite side shows much more regular and sharper ribs than usually would be expected at the dorsal side. No traces of a ligamental crest could be found!! On the other hand some traces of the dentition have been found (f. 2). If our interpretation of these traces is correct, the animal must have had well developed myophores as in *Radiolitinae* and *Petalodontiae*. (It must be stated, however, that in a higher section these traces of sockets and myophores can no more be seen, because of the bad preservation of the inner shell layer.) The anterior myophore of the lower valve is rather small and notched exteriorly. The position of the siphonal zone fits well into this interpretation.



Fig. 3.
Caprinuloidea perfecta 1/1.

For explanation see fig. 4.

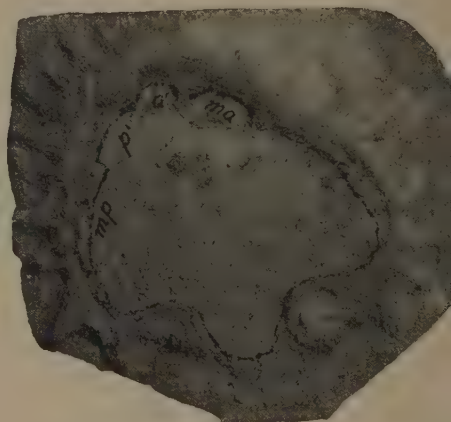


Fig. 2.
? *Tepeyacea corrugata* 1/1.

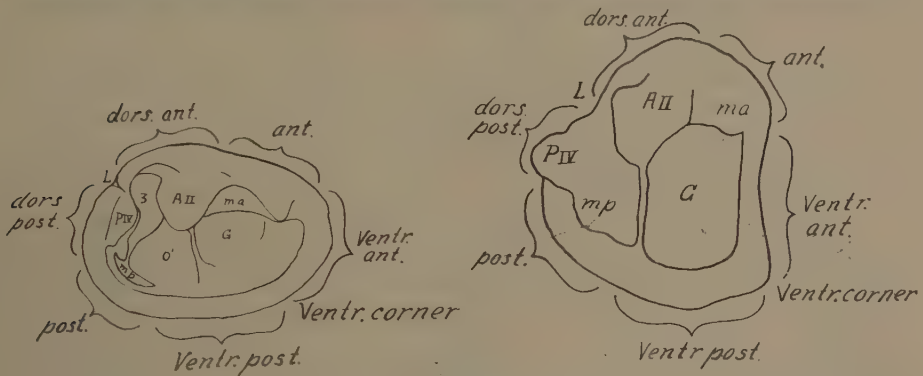
Our material is so like PALMER's *Tep. corrugata*, that we are inclined to identify it with this species, if only to avoid the creating of yet another genus. PALMER, however, mentions the existence of a distinct ligamental "fold or ridge extending down the inner side of the outer layer and imbedded in the inner layer". His figure, however, is far from convincing with regard to this point. The myophores are supposed by PALMER to be superficial, because he found no trace of them, which is a poor argument. The inner shell layer of the cuban specimens seems to be thinner than in the

mexican ones. Should the existence of a ligamental crest in the mexican form be proved, then the cuban form would belong to a new genus, differing from *Tepeyacia corrugata* by the absence of a ligamental crest. The compact lamellar structure together with the absence of a ligamental crest precludes, as far as we know, its belonging to any known genus. On the whole we think our cuban form more closely related to the *Radiolitinae* than to the *Monopleurinae*, because of the nature of the cardinal apparatus and the absence of a ligamental crest.

Caprinuloidea perfecta PALMER [f. 3, 4 (1—8), 5 (8)]. Occ. Papers Cal. Acad. Sc. XIV. 1928, p. 59—60, pl. VIII. f. 2, IX. f. 1. 2; textf. 6.

Very common at A236. In one large block we found four specimens with both valves partly or wholly preserved, moreover one lower and two upper valves.

External features: Shell large; surface, so far as to be seen on tangential and horizontal sections, smooth; no growth-layers, neither longi-



Two sketch figures to show the difference of orientation of *Caprina* and *Caprinuloidea*.

Left fig. *Caprina adversa* (after DOUVILLÉ 1936. B.S.G.F. Page 340, fig. 13).

Right fig. *Caprinuloidea perfecta* (after PALMER Occ. Papers Cal. Acad. of Science, fig. 6, pag. 54).

tudinal riblets. Upper valve thoroughly coiled towards the anterior, forming a loose spiral, concave at dorsal side. Moreover it is torded in the same direction. Upper valve at least 20 cm along the ventral side. Lower valve straight or almost straight, measuring at least 13 cm. Internal ligament marked on the surface by a groove. Ventrally the shell has a conspicuous rib in both valves. In section near the commissure the shell is triangularly oval, in diameter antero-posteriorly \times dorso-ventrally about 7×5 cm. **Internal features of the upper valve:** Shell composed of three layers, a compact dark thin outer one, a broad middle and a compact thin inner one. The middle layer is composed of vertical walls, forming longitudinal canals. These canals are not tabulate. At the dorsel side anteriorly of the ligament the canals are formed by vertical, radial walls, sometimes with some trans-

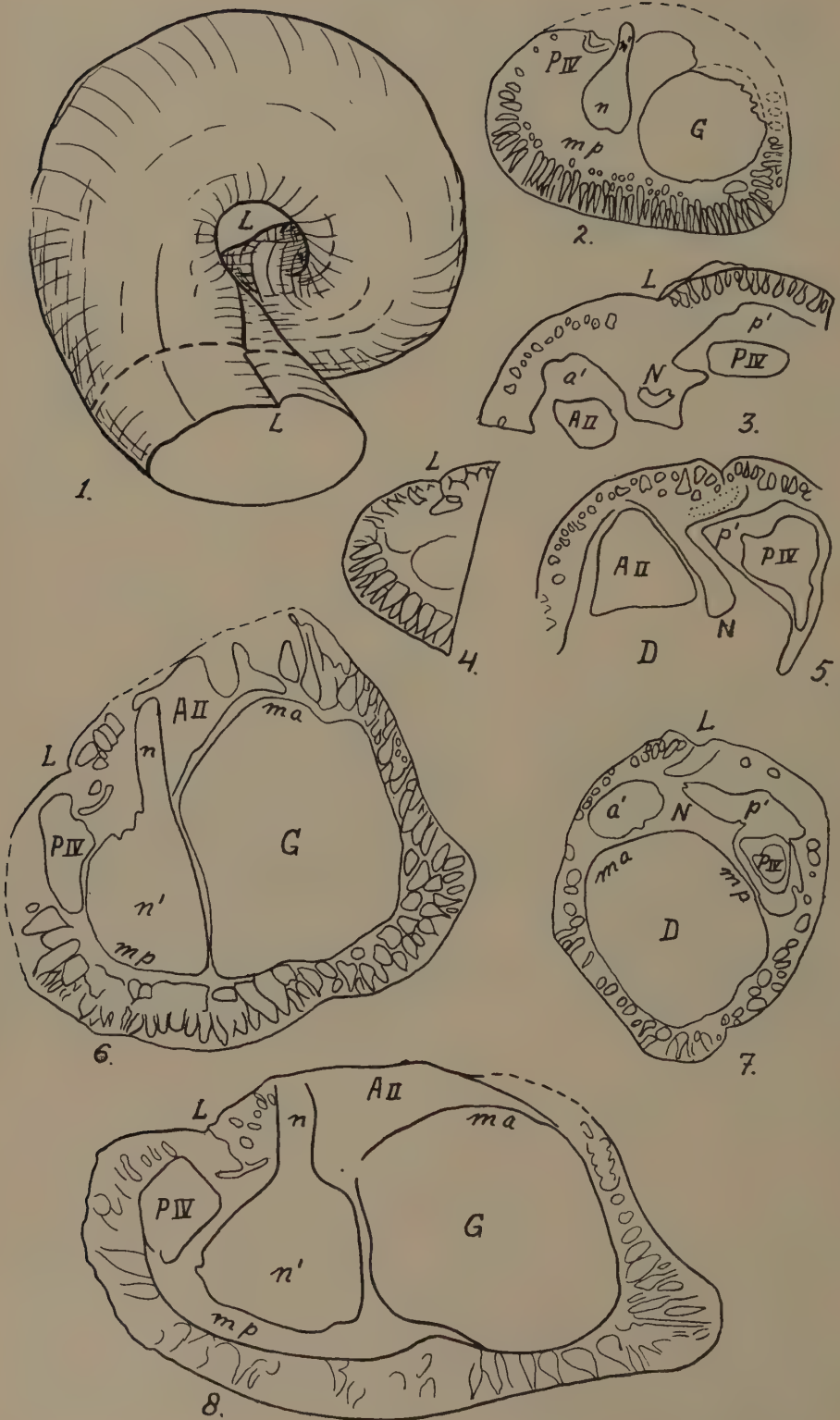


Fig. 4.

EXPLANATION OF FIG. 4.

1—8. *Caprinuloidea perfecta* PALMER. 1: Reconstruction projected by H. J. MAC GILLAVRY $\times 0,45$. 2: Section of separate upper valve. Dorsal side is not preserved. 3: Part of lower valve of the same specimen as 8. 4: Part of upper valve of young animal. 5: Lower valve of same specimen as 4. 6: Section of separate upper valve. 7: Section of separate lower valve showing the oblique situation of the *PIV*. 8: Section of upper valve of the same specimen as 3. *L* ligament, *N* the tooth of the lower valve, *A II* anterior and *P IV* posterior teeth of the upper valve, *ma* and *mp* the anterior and posterior myophores, *G* and *D* the body cavity in upper and lower valve, *n'* posterior accessory cavity in the upper valve, *n* the tooth alveole in the upper valve, *a'* anterior, *p'* posterior tooth alveole of lower valve. All figures, except no. 1, are of natural size. All specimens from A 236.

verse tangential walls, thus being radial or polygonal. Posteriorly of the ligament on the dorsal side the canals may be radial or pyriform. In the dorsal anterior corner sometimes the middle layer is thick, up to 1 cm, and then there are marginal pyriform canals and inside of these roundish polygonal ones. If this anterior corner is not thicker than the adjacent parts, there is a single or double row of pyriform canals formed by resp. single or bifurcating vertical radial walls, sometimes with tangential walls forming polygonal canals. On the ventral anterior side there may be one or two rows of pyriform canals. The ventral corner is always thick and has one or two rows of pyriform canals and centrad from these sometimes polygonal ones. Ventral-posteriorly there are two rows of pyriform canals, with in the larger ones often tangential walls, forming polygonal canals. On the posterior side there are one or two rows of pyriform canals with sometimes vertical transverse walls. The ligament is internal. The ligamental infolding has an anchor-shaped section and is $\frac{1}{2}$ — $\frac{3}{4}$ cm deep. The body cavity is round on section. It is separated by a thin septum from the large posterior accessory cavity *n'*, which sometimes shows small constrictions [fig. 4 (6, 8) fig. 5 (8)], *n'* is connected with the toothsocket *n*. *n* is septate, that means subdivided by vertical septa or walls at the bottom. Remarkable is the oblique position of the cardinal apparatus with regard to the ventral side, the *mp* lying almost in the prolongation of the ventral side. In correspondence with this the entire section is triangularly oval and the *n'* is broad on the ventral side. *PIV* is robust, always situated closely behind the ligament; *mp* long, extended along *n'*, even surpassing a little the septum between *n'* and *G*, reaching the posterior ventral corner of the body cavity *G*. *A II* is robust too; *ma* is a prolongation of *A II*, smaller than *mp*.

Internal features of the lower valve. Shell structure and ligament as in the upper valve. Canals around the whole circumference. Dorso-anteriorly of the ligament there are pyriform canals with some transverse vertical walls or polygonal canals. Posteriorly of the ligament canals pyriform or radial. On the anterior side pyriform and radial, at the thick ventral corner pyriform and polygonal canals. Posterior-ventrally one, two or three rows of pyriform canals and sometimes polygonal ones, on the posterior side a single or double row of pyriform canals with tangential walls or polygonal canals. Shell tripartite: body cavity round, posterior tooth alveole connected to the

ventral side with a large posterior accessory cavity, separated from the body cavity by a thin septum, the mp., which projects from the large

Table showing the canal pattern in several specimens of *Caprinuloidea perfecta* (A236)

UPPER VALVE						
Specimens	A	B	C	D	E	M
Dors.Post.	pyr.	?	pyr-rad.	-	rad.	pyr & rad.
Dors.Ant.	pol-rad.	rad-pol.	rad.	-	rad.	rad.
Ant.	rad.	2 pyr & tr.	2 pyr-rad.	-	pyr & tr.	?
Ventr.Ant.	1-2 pyr.	2 pyr.	1 pyr.	-	1 pyr.	pyr & pol.
Ventr.Cor- ner.	2 pyr & pol.	2 pyr & pol.	2 pyr & pol.	-	2 pyr & pol.	pyr-pol.
Ventr.Post.	pyr & tr.	2 pyr & tr.	2 pyr.	-	pyr & tr.	pyr-pol.
Post.	2 pyr.	2 pyr & tr.	2 pyr.	-	pyr & tr.	2 pyr & tr- pol.
LOWER VALVE						
Dors.Post.	-	-	pyr.	pol.	rad.	?
Dors.Ant.	-	-	rad.	rad.	pyr & pol.	?
Ant.	-	-	-	rad-pol.	pol.	pyr & rad.
Ventr.Ant.	-	-	-	1 pyr.	pyr & pol.	rad.
Ventr.Cor- ner.	-	-	-	1 pyr & pol.	-	pyr & pol.
Ventr.Post.	-	-	-	pyr & pol.	pyr & pol.	pyr & pol.
Post.	-	-	1-2 pyr.	pol.	2 pyr & tr.	pyr & pol.

Explanation: pyr is pyriform, 1 pyr is a single row of pyriform canals etc.; pyr & tr means pyriform canals with transverse tangential walls; rad. is radial canals; pyr-rad means pyriform to radial canals; pyr & rad. means pyriform and radial canals; pol. is polygonal canals; pol-rad. is polygonal to radial canals; — means not preserved; ? preserved but indistinct. B is f. 4 (6); C is f. 4 (3, 8); D is f. 4 (7); E is f. 4 (4, 5).

quadrangular tooth N, anterior tooth alveole equally separated from the body cavity by a septum, also projecting from the N. In one specimen PIV is thrust obliquely into its alveole, directed towards the ventral side, into the accessory cavity.

This form is a *Caprinuloidea* for the following reasons: upper valve bipartite, lower valve tripartite, canals around the whole circumference in both valves, pyriform and polygonal, ligament internal. *Schiosia* always has a single row of pyriform canals and polygonal ones only on the anterior side. *Offneria* is not tripartite in the lower valve and has a different canal pattern (many large, polygonal canals with thin walls). *Coalcomana*, closely related, as PALMER mentioned never has polygonal canals on the posterior and ventral side. This species is clearly different from all other species of the genus by being robust, having pyriform canals, which are not tabulate, a smooth surface, and only one "sulcus" on the surface. The cuban form differs slightly from the mexican one in the canal pattern (the polygonal canals are less in number and more roundish in shape) and the form of the upper valve. These differences however are not essential.

Caprinuloidea sp. [f. 5 (2, 3)].

From A236 and L328 there are two fragments of upper valves which

show the generic features of *Caprinuloidea*. Upper valve loosely coiled. Surface not visible. Diameter antero-posteriorly \times dorso-ventrally 5.5×4 and 4.5×3.5 cm. Ligament interior. Canals on the whole circumference. Dorsally, posterior of the ligament a single pyriform marginal row, posteriorly a single or double row of pyriform canals, with sometimes transverse vertical walls in the larger ones, and here and there typical polygonal canals. Ventral-posteriorly as on the posterior side. On the ventral anterior side a single row of radial canals. In the anterior ventral corner a marked rib with two rows of pyriform and one of polygonal canals centrad of them. Dorsally anterior of the ligament radial canals; on the anterior side radial canals too. The round body cavity separated by a thin septum from a large, ventrally broad posterior accessory cavity n' , connected with the tooth alveole n , which is subdivided by vertical walls at the bottom. Dental apparatus as in *C. perfecta*.

This species differs from *C. perfecta* by the presence of a marked anterior-ventral rib and by the pyriform canals being longer and thinner. As nothing is to be seen of the outer surface and the lower valve is unknown, the material is too poor for adequate specific determination and description.

Coalcomana ramosa (BOEHM). f. 5 (4—7). HARRIS and HODSON, Pal. amer. 1. 3. 1922. p. 14 (with previous literature); — PALMER. Occ. Papers Cal. Acad. Sc. XIV. 1928. p. 69. pl. XII. f. 3, 4.

From M 656 fragments of two upper valves and two lower valves; from L 58 and L 59 fragments of upper valves. Upper valve: one of the M 656 fragments has the common canal pattern as in the fig. of HARRIS and HODSON. Ventrally, the vertical radial walls are twice branched, thus forming three rows of pyriform canals. In the L 59 and the other M 656 fragments these ventral canals are very slender. [See f. 5 (4, 7).] The two lower valves show the same aspect as PALMER's photographs of the specimen from Soyatlan de Adentro, Jalisco. Canals around the whole circumference, polygonal only anteriorly and dorsally, shell tripartite, ligament internal, marked on the surface by a groove. These valves, however, differ in three respects from the type specimens from Coalcoman described and figured by HARRIS and HODSON and H. DOUVILLÉ.

Coalcomana ramosa from Coalcoman

Several roundish polygonal canals only near the ma.

mp long, nearly parallel to the posterior side

No ligamental groove on the surface

Coalcomana ramosa from Soyatlan de Adentro, Jalisco and Cuba.

Many large and small polygonal canals along the whole anterior side.

mp shorter, directed toward the posterior side.

A marked ligamental groove on the surface.

BOEHM's specimen also had a ligamental groove, PALMER's specimen shows still more polygonal canals than our.

Sabinia sp. [f. 5 (1)].

A large fragment of an upper valve in solid rock from L328. It is rather slightly coiled, dorsally concave. It has a very peculiar elongate

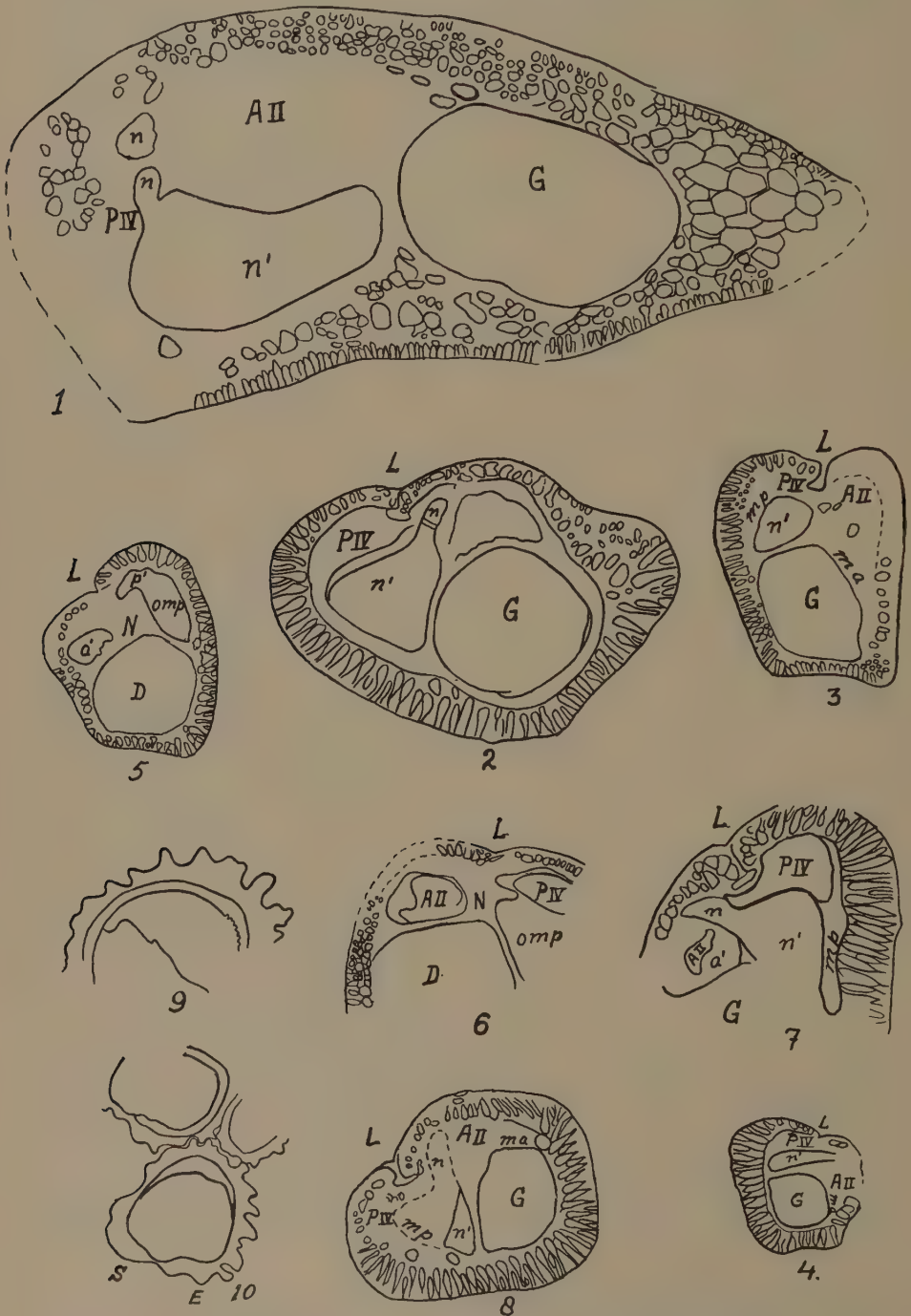


Fig. 5.

EXPLANATION OF FIG. 5.

For explanation of the letters see fig. 4. All figures nat. size.

1. *Sabinia* sp. from L 328.
2. *Caprinuloidea* sp. from A 236.
3. *Caprinuloidea* sp. from L 328.
- 4—7. *Coalcomana ramosa* (G. BOEHM) 4: upper valve with long slender pyriform canals on the posterior and ventral side; from M 656. 5 and 6: lower valves from M 656.
- 7: upper valve from L 59, showing long pyriform canals at the posterior side.
8. *Caprinuloidea perfecta* from V 68. This form belongs to this species because of the three polygonal canals near the *mp*.
- 9, 10. ? *Tepeyacia corrugata*. PALMER. From L 328.

section, diameter measuring dorso-ventrally \times antero-posteriorly 12×5 cm. Dorsal side somewhat broken; ligament not visible. Therefore we are not sure that this fragment belongs to the genus *Sabinia*. There are canals all around the circumference. Dorsally nothing can be seen exteriorly, but more to the interior some irregular rows of roundish polygonal thickwalled canals exist. At the posterior side outside a 2 mm broad marginal row of pyriform canals and centrad of this a 7 mm thick layer of polygonal canals of much varying width (1—4 mm). On the ventral side a marginal row of pyriform canals and centrad of it at the corner an at least 20 mm broad layer of polygonal canals, of 7 mm. Anteriorly an irregular, marginal row of polygonal canals and a 6 mm broad layer of somewhat larger canals centrad of it. Canals tabulate, the tabulae about 1 cm apart. Accessory posterior cavity *n'* large and connected with the tooth alveole *n*, which is subdivided by vertical septa. The septum separating *n'* and *n* from *G* very thick and short. PIV small, AII large. Myophores cannot well be distinguished.

This form is clearly distinct from *Caprinuloidea multitubifera* and *Coralliochama* by its irregular and sometimes wide spaced canals. It differs from the other species of *Sabinia* by its oblong section and its canal pattern.

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Anatomy. — *Die Pyramidenbahnen von Echidna.* Von J. L. ADDENS und TOSHIYUKI KUROTSU. (Aus dem Niederländischen Zentralinstitut für Hirnforschung, Amsterdam). (Communicated by Prof. C. U. ARIËNS KAPPERS).

(Communicated at the meeting of October 31, 1936).

Es war ein ganz zufälliger Umstand, der uns auf den Gegenstand vorliegender Mitteilung führte. Als nämlich der jüngere Autor gelegentlich anderer Untersuchungen nicht nur die ihn gerade beschäftigenden Bildungen in seinen Figuren bezeichnen wollte, geriet er in Zweifel, welches Faserbündel bei *Echidna* als Pyramide zu deuten wäre. Da die Literatur, wie aus der folgenden Uebersicht hervorgehen wird, nur sich widersprechende Angaben enthält, entschlossen wir uns die Sache selbst zu untersuchen.

Von älteren Autoren haben ZIEHEN, KÖLLIKER und ELLIOT SMITH Angaben über die Pyramiden von *Echidna* gemacht. ZIEHEN (1897) und ELLIOT SMITH (1902) behaupten sie schon bei äusserer Betrachtung als breite, flache Bänder an der Basis der *Oblongata* gesehen zu haben.

Der erste, der die Pyramiden von *Echidna* und ihre Kreuzung mikroskopisch untersuchte, war ZIEHEN (1897a, '99, 1908), der dabei aber zu keinen bestimmten Ergebnissen kam. Nach ihm findet sich unmittelbar neben der Raphe eine Zone sehr feiner, ziemlich zerstreuter, blasser Fasern, die vielleicht als Pyramidenbahn aufzufassen wären. Eine kompakte Pyramidenkreuzung fehlt. Statt dessen sollte sich an der gewöhnlichen Stelle, am Anfang des Rückenmarks, eine ganz zerstreute Kreuzung feiner Faserbündelchen in der Raphe vorfinden. Die gekreuzten Fasern sollten meistens, wenn nicht alle, zu den Seitensträngen ziehen. KÖLLIKER (1901) gelangte zu ähnlichen Ergebnissen.

Von neueren Autoren haben sich FUSE (1926a) und ABBIE (1934) mit den Pyramidenbahnen bei *Echidna* beschäftigt, ersterer in einer besonderen Abhandlung, letzterer in seiner ausführlichen Monographie über den Hirnstamm und das Cerebellum dieses Tieres.

Von FUSE wird das Vorkommen einer Pyramide an der gewöhnlichen Stelle, ventral vom Lemniscus medialis, wie es ZIEHEN und KÖLLIKER annehmen, völlig verneint. Die von diesen Autoren behauptete Kreuzung soll lediglich ein Fasersystem der Commissura ventralis, wie es auch den übrigen Säugern zukomme, darstellen. Dagegen wird von FUSE eine ganz andere Bahn, nämlich die von KÖLLIKER (1901) entdeckten und als der Zonalbogen oder die Zonalbündel bezeichneten Fasern, als Pyramide angesprochen. Es sind dies Fasern, die in der *Oblongata* die einwärts

gekrümmte absteigende V-Wurzel wie eine Schale umfassen, wobei sie sich durch ihre dunklere Färbung von letzterer abheben (Fig. 5, pyr. lat.).

In der Brücke kreuzen sich die Zonalbündel, nachdem sie sich von der V-Wurzel getrennt haben, und begeben sich nach KÖLLIKER in den *Pes pedunculi*, den *Lemniscus medialis* und den *Colliculus anterior*. KÖLLIKER neigt am meisten dazu, diese Fasern für eine spinocerebrale Bahn, eine aufsteigende sensible Bahn zweiter Ordnung also, zu halten. Weder beim *Ornithorhynchus* noch bei irgend einem anderen Säugetier konnte KÖLLIKER eine Spur solcher Fasern auffinden. ZIEHEN (1908) hat diese Angaben KÖLLIKERS völlig bestätigt.

Im Gegensatz zu FUSE kehrt der neueste Untersucher des Echidnagehirns, ABBIE (1934), bezüglich der Pyramiden in der Hauptsache zu den Auffassungen ZIEHENS und KÖLLIKERS zurück, indem er das Vorkommen zwar kleiner, aber typischer Pyramiden mit Kreuzung an der gewöhnlichen Stelle annimmt. Nach ABBIE aber enden die Pyramidenfasern unmittelbar an den motorischen Vorderhornzellen. Die Deutung der Zonalbündel von FUSE als Pyramide weist ABBIE zurück. Nach ihm entsteht diese Bahn, die er *Tractus temporo-trigeminalis* nennt, aus derselben Hirnregion wie die temporo-pontinen Fasern (Rinde vom Typus IV SCHUSTERS, 1919) und hat daher mit einer Pyramide nichts zu tun.

Wir schreiten jetzt zu unseren eigenen Beobachtungen und zur kritischen Besprechung der Befunde und Ansichten unserer Vorgänger.

Für unsere Untersuchung benutzten wir hauptsächlich eine transversale WEIGERT-PAL-Serie, nachgefärbt mit Parakarmin. Dieser Serie sind sämtliche Abbildungen entnommen. Ausserdem wurden zum Vergleich herangezogen zwei transversale WEIGERT-Serien, wovon die eine dem Institut geschenkt wurde von Prof. J. SYMINGTON, während die andere sich früher im Besitze von Prof. TH. ZIEHEN befand, und schliesslich eine transversale Hämatoxylin-Serie. Alle diese Serien entstammten der Form des Australischen Kontinents, *Echidna aculeata* Shaw.

Wir wollen uns zuerst mit der Frage befassen, ob sich bei *Echidna* eine Pyramide an der gewöhnlichen Stelle, ventral vom *Lemniscus medialis*, findet (ZIEHEN, KÖLLIKER, ELLIOT SMITH, ABBIE) oder ob sie dort fehlt (FUSE).

Die Durchsicht der WEIGERT-PAL-Parakarmin-Serie ergab sofort, dass ein feinfaseriges, schwach markhaltiges Bündelchen an der in Frage kommenden Stelle unzweifelhaft anwesend ist (Fig. 4 und 5: pyr. med.). Abgesehen von seiner geringen Grösze macht das Bündel ganz den Eindruck einer typischen Pyramide. Am deutlichsten ist es im Hinterende des Pons und besonders in dem diesem unmittelbar folgenden Teil der *Oblongata*. In allen Serien ist es aber nicht gleich leicht aufzufinden.

Am bequemsten war dies der Fall in der WEIGERT-PAL-Parakarmin-Serie, wo die Bündel ventral vom *Corpus trapezoides* in einem faserfreien Gebiete liegen, wie es schon von ABBIE beschrieben wurde. Die Hämatoxylin-Serie zeigte dieselben Verhältnisse.

In den SYMINGTON- und ZIEHEN-Serien aber wurden die Pyramiden von

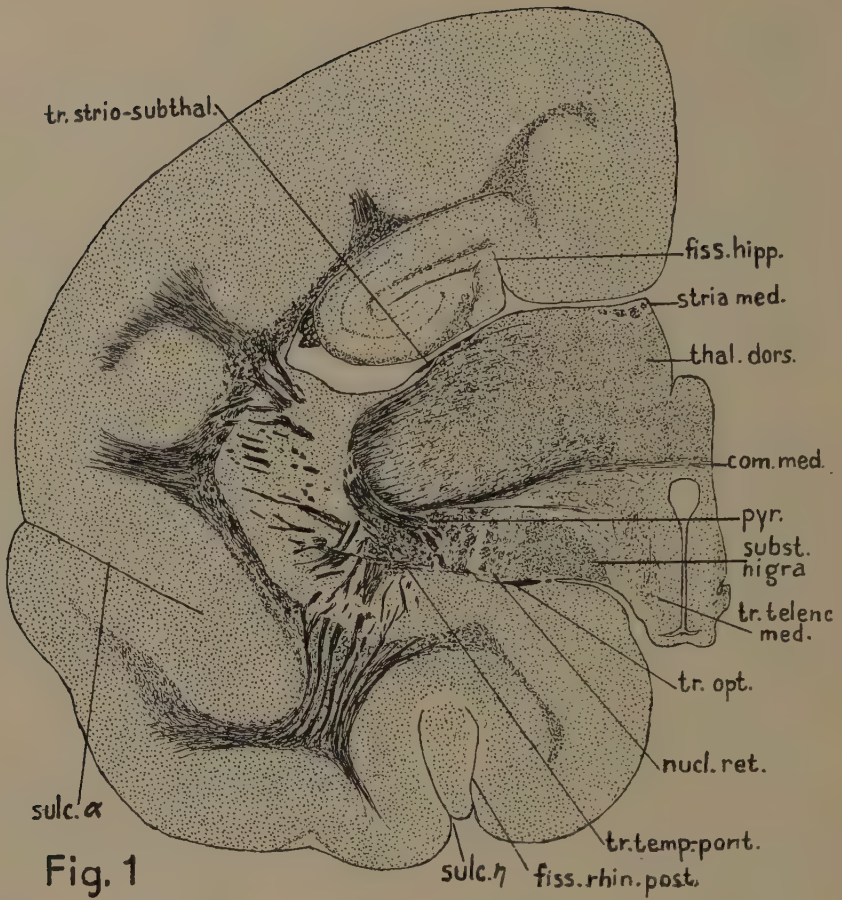


Fig. 1

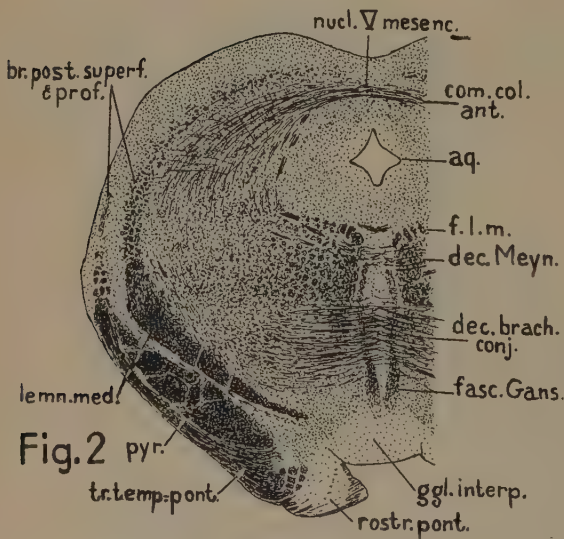


Fig. 2

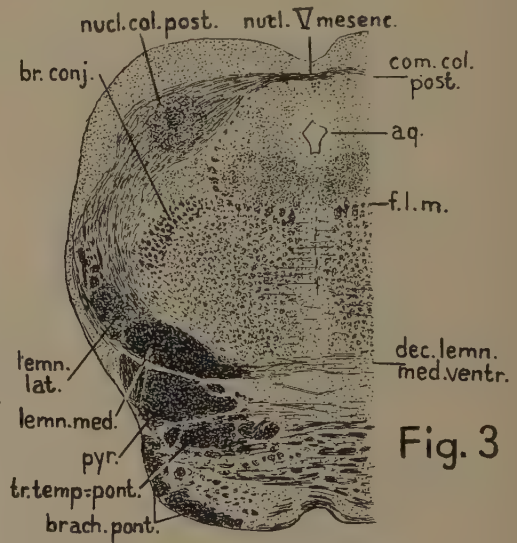


Fig. 3

Fig. 1—3. *Echidna aculeata*. Querschnitte durch Thalamus und Hemisphäre (1), den Anfang des Rostrum pontis (2) und etwas weiter kaudal durch das letztere (3). WEIGERT-PAL-Parakarmin. Fig. 1: $\times 3\frac{1}{2}$; Fig. 2 und 3: $\times 6$.

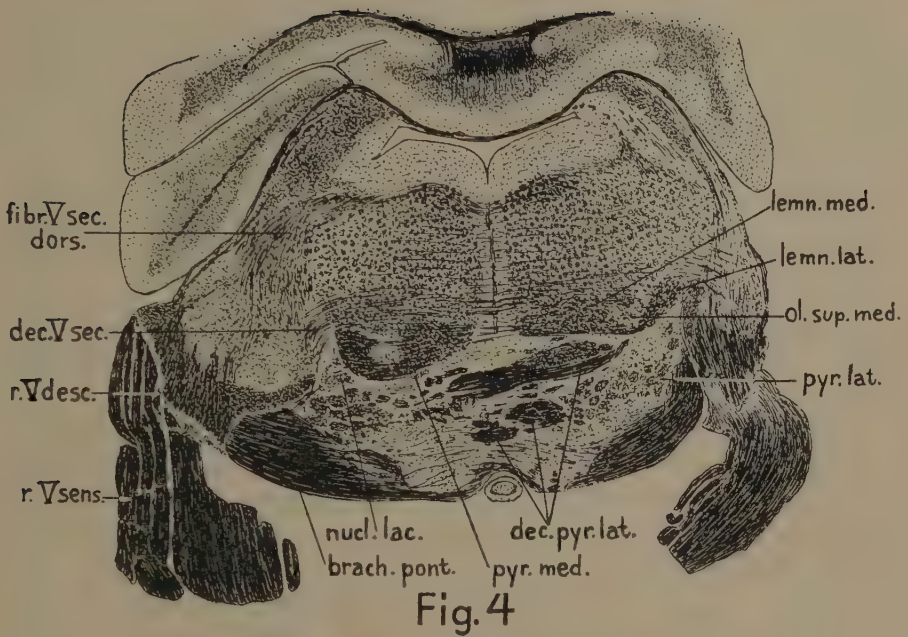


Fig. 4

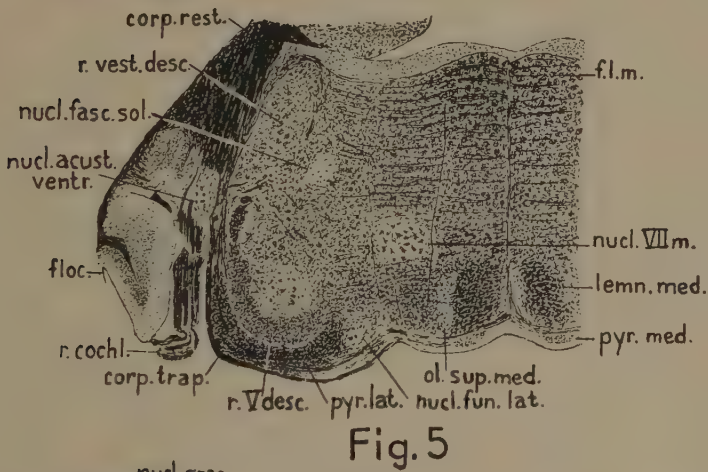


Fig. 5

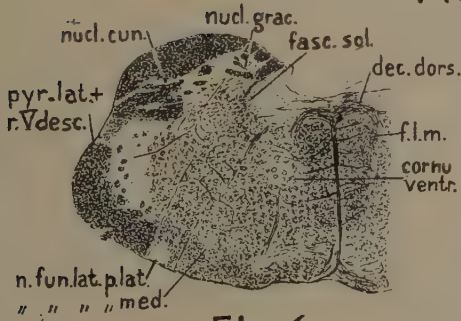


Fig. 6

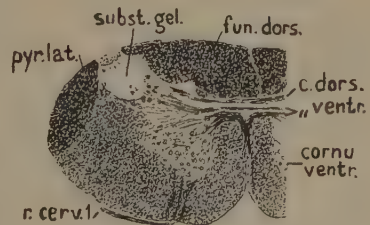


Fig. 7

Fig. 4—7. *Echidna aculeata*. Querschnitte in der Höhe des Endes des Pons (4), des Endes des Eintritts der Cochleariswurzel (5), des Anfangs des Vorderhorns (6) und des Austritts der ersten motorischen Cervicalwurzel (7). WEIGERT-PAL-Parakarmin. Alle Figuren $\times 6$. Nucl. lac. (in Fig. 5) = Nucleus laciniatus.

den Trapezfasern durchquert und zerklüftet oder lagen sogar dorsal von denselben, wodurch das Bild sehr getrübt wurde. Wahrscheinlich haben ZIEHEN, KÖLLIKER und FUSE allein derartige Serien vorgelegen, wodurch sich die Unbestimmtheit in den Äusserungen ZIEHENS und KÖLLIKERS und das völlige Verneinen des Vorkommens einer Pyramide an der gewöhnlichen Stelle seitens FUSES erklärt.

So ist ABBIE der erste gewesen, der bei *Echidna* eine typische Pyramide mit Sicherheit festgestellt hat.

Wie eingangs erwähnt, behaupten ZIEHEN (1897) und ELLIOT SMITH (1902) die Pyramiden schon äusserlich als breite, flache Bänder an der Basis der Oblongata gesehen zu haben. Ohne Zweifel haben sie sich dabei getäuscht. Denn, wie das mikroskopische Präparat zeigt, sind die Pyramiden ganz schmal. Ob ABBIE meint diese Bündel wirklich makroskopisch gesehen zu haben oder nur schematisch die Stelle ihres Vorkommens in seiner Figur der Unterseite des Hirnstammes angibt, ist uns nicht klar geworden. An einigen in Formol konservierten, dem Institut von Prof. BURKITT, Sydney, geschenkten Gehirnen, ist es uns auch mit der Binokularlupe nicht gelungen die Pyramiden zu Gesicht zu bekommen.

Obleich wir also bezüglich der Anwesenheit einer Pyramide im Pons und dem ihm unmittelbar folgenden Teil der Oblongata völlig mit ABBIE einig sind, weichen unsere Beobachtungen über den Rest ihres Verlaufs, sowohl nach vorne wie nach hinten, von den seinigen ab.

ABBIE gibt folgende Beschreibung des Verlaufs der Pyramiden. Sie gehen hervor aus der vordersten Gegend der Hemisphären (Rinde vom Typus III SCHUSTERS, 1910) und sind in der medialen Ecke des *Pes pedunculi* als ein schwaches Bündel feiner, leicht gefärbter Fasern, lateral von der Substantia nigra zu unterscheiden. Im Pons gehen sie sofort zur ventralen Seite des Lemniscus medialis und bleiben dort während ihres weiteren Verlaufs in der Oblongata. Schon in der Oblongata kreuzt ein Teil der Fasern und begibt sich zu den motorischen Hirnnervenkernen. Am Anfang des Rückenmarks tritt aber die Hauptkreuzung auf, deren Fasern nach ABBIE unmittelbar an den Vorderhornzellen enden.

Wir haben uns die grösste Mühe gegeben das in Frage kommende Bündel so genau wie möglich zu verfolgen. Wie gesagt, ist es am deutlichsten im Hinterende des Pons und dem ihm unmittelbar folgenden Teil der Oblongata. Nach vorne konnten wir seinen Ursprung nicht mit Sicherheit ermitteln. Am wahrscheinlichsten zweigt es sich am Anfange des Pons vom gleichseitigen *Pes pedunculi* ab, ist also ein ungekreuztes Bündel. Es war uns jedoch unmöglich das Bündel wie ABBIE im *Pes pedunculi* als einen besonderen Anteil desselben zum Vorderhirn zu verfolgen. Was aber die Natur der Fasern ist, die ABBIE in der medialen Ecke des *Pes pedunculi* als Pyramide bezeichnet, können wir nicht entscheiden.

Auch bezüglich der hinteren Strecke des Verlaufs der Pyramiden weichen wir, wie erwähnt, von ABBIE ab. Beim rückwärts Verfolgen des Bündels verloren wir es in der Höhe des oralen Endes des XII-Kernes aus dem

Auge. Dasz die von ZIEHEN, KÖLLIKER und ABBIE am Anfang des Rückenmarks beschriebene Kreuzung aus diesem winzigen, aus feinen, schwach markhaltigen Fasern zusammengesetzten Bündelchen hervorgeht, erscheint uns ausgeschlossen. Ist doch die Kreuzung ansehnlich uns aus groben, gut myelinisierten Fasern zusammengesetzt. Wir pflichten denn auch FUSE (1926a, p. 98; vgl. 1926, p. 76—78) bei, der, wie erwähnt, diese Fasern als zur *Commissura ventralis* gehörig betrachtet.

ABBIE erachtet die Pyramide bei *Echidna* der direkten oder ventralen Pyramide des Menschen homolog. Diese verläuft, wie bekannt, im Vorderstrang, während ihre Fasern erst nach und nach, in der Höhe ihrer Endigung im gegenüberliegenden Vorderhorn, zur anderen Seite ziehen. Wir wollen nicht behaupten, dasz diese Deutung unmöglich sei, auch für andere niedere Säuger ist eine direkte Pyramide beschrieben worden, so z.B. für den Maulwurf, wo die ganze Pyramidenbahn im Vorderstrange nach hinten zieht. Weil wir aber das betreffende Bündel in der Höhe des oralen Endes des XII-Kernes aus dem Auge verloren, mag es sich lediglich um eine cortico-bulbäre Bahn handeln. Nur degenerative Untersuchungen können hier entscheiden.

Wir wenden uns jetzt den sogen. Zonalbündeln KÖLLIKERS zu. Wie schon hervorgehoben, werden sie von ihrem Entdecker, ZIEHEN und ABBIE, der sie zum *Tractus temporo-trigeminalis* umtaufte, als eine Bahn betrachtet, die unter allen Säugetieren bis jetzt nur bei *Echidna* wahrgenommen sei. Wo es sich um einen so mächtigen Faserzug handelt, scheint diese Ansicht schon von vornherein wenig wahrscheinlich.

Der einzige Autor, der dieses Bündel als eine der den Säugern allgemein zukommenden Bahnen und zwar als Pyramide deutete, war, wie erwähnt, der japanische Forscher FUSE (1926a), der sich in mehreren Abhandlungen (1926, '26a, b und c) sehr verdient gemacht hat um die vergleichende Anatomie der Pyramiden.

Der Verlauf des Bündels, soweit dieser bis jetzt bekannt war, ist in der Hauptsache folgender. Es geht am Anfange des Pons aus dem *Pes pedunculi* hervor (Fig. 2 und 3: pyr.), schmiegt sich, nachdem es gekreuzt hat, der medialen Seite der absteigenden V-Wurzel an (Fig. 4: pyr. lat.) und schiebt sich allmählich zwischen dieser und dem *Brachium pontis* ein, so dasz es die absteigende V-Wurzel schliesslich wie eine Schale umfasst. Durch seine dunklere Färbung hebt es sich von dieser ab. Sich allmählich am Rande der *Oblongata* hinaufziehend (Fig. 6) ziehen dann beide Bündel zusammen zum Rückenmark, wo sie den dorsalsten Teil des Seitenstranges bilden (Fig. 7).

Die Deutung FUSES beruht darauf, dasz die KÖLLIKERSchen Zonalfasern eine kreuzende Verbindung darstellen zwischen dem Vorderhirn und dem Seitenstrang, namentlich dessen Pyramidengebiet. Als Stütze seiner Auffassung führt FUSE an, dasz bei mehreren Tieren die Pyramiden sich schon weit nach vorne, in der Gegend des VII-Kernes, kreuzen. Längst bekannt ist dies Verhalten von den Fledermäusen, wo es 1903 gleichzeitig von

DSÄSEKE und HATSCHEK entdeckt wurde. FUSE (1926, 1926c) selbst fand es ausserdem bei einigen Edentaten. Hier können sogar die gekreuzten Pyramidenfasern sich der absteigenden V-Wurzel ventromedial anschliessen. Auch der Elephant zeigt eine derartige vordere Pyramidenkreuzung (BREGMANN, 1915). Weiter führt FUSE das abnormale Pyramidenbündel „a“ von KARPLUS und SPITZER (OBERSTEINERS Arbeiten, Bd. 11, 1904) beim Menschen an, das der ventrolateralen Seite der absteigenden V-Wurzel anliegt.

Wie schon hervorgehoben, gibt ABBIE an, dass die Fasern der sog. Zonalbündel mit den temporo-pontinen Fasern zusammen aus der Rinde vom Typus IV SCHUSTERS entstehen, was der Deutung als Pyramide widerspricht. FUSE sagt nur, dass das Bündel aus dem Pes pedunculi hervorgeht.

Wir haben daher die Entstehung des Pes aus der Rinde nachgeprüft und fanden dabei (Fig. 1), dass die laterale Hälfte des Pes gebildet wird von Fasern, die ventral und kaudal vom Sulcus α ELLIOT SMITHS¹⁾ entstehen, d.h. aus Rinde vom Typus IV SCHUSTERS. Die mediale Hälfte dagegen geht hervor aus derben Faserzügen, die dorsal und oral von dieser Furche entstehen, aus Rinde vom Typus III.

Weiter nach hinten sind beide Anteile des Pes nicht mehr von einander zu unterscheiden. Beim Eintritt in den Pons aber ziehen die dorsolateralen Fasern des Pes an der Auszenseite der medialen entlang ventralwärts (Fig. 2) und bilden ein Bündel (Fig. 3), das sich im Pons auflöst. Dies wird also der Tractus temporo-pontinus sein. Der übrig gebliebene, gleich starke Teil kreuzt im Pons und ist die Pyramide.

Wir möchten an dieser Stelle noch einige Bemerkungen über die lateralen Pyramiden hinzufügen. Erstens über den Färbungsunterschied im Vergleich zur absteigenden V-Wurzel. Die dunklere Färbung der Pyramiden wird hauptsächlich dadurch hervorgerufen, dass ihre Markscheiden sehr viel stärker entwickelt sind. Nach ZIEHEN (1908) sind aber die Achsenzyylinder der Pyramidenfasern feiner als die der V-Wurzel. Bei einem jugendlichen Tiere fand ZIEHEN den Färbungsunterschied gerade umgekehrt, die absteigende V-Wurzel war die dunklere, was durch die spätere Markentwicklung in der Pyramide verursacht wird.

Unsere Beobachtungen über die Art und Weise der Kreuzung der lateralen Pyramiden weichen von denjenigen unserer Vorgänger nicht unerheblich ab. Vielleicht sind hier aber individuelle Variationen im Spiel.

Nach KÖLLIKER (1901) gehen seine Zonalbündel im Rostrum pontis, dem schnabelförmigen vorderen Fortsatz des Pons, eine diffuse Kreuzung ein, untermischt mit Brückenarmfasern. ZIEHEN (1908) hat dies bestätigt. Nach FUSE (1926a) kreuzen die Pyramiden sich in diffuser Weise in der Sagittaltmitte der ventralen Brücke.

In der WEIGERT—PAL-Parakarmin-Serie, die dieser Mitteilung haupt-

¹⁾ Siehe für eine eingehende Beschreibung der Furchen des Echidnagehirnes und Erörterung ihrer Nomenklatur die Arbeit BURKITTs (1934).

sächlich zu Grunde liegt, kreuzen die lateralen Pyramiden sich aber im dorsalen Teile des Hinterendes des Pons, wobei sie sich nicht verflechten, sondern als geschlossene Bündel über einander hinziehen (Fig. 4). Diesem Umstande zufolge war hier der ganze Verlauf der Pyramiden im Pons einwandfrei zu verfolgen. Auch ABBIE hat auf Grund derselben Serie die Kreuzung seiner Tractus temporo-trigeminales (unserer lateralen Pyramiden) im Hinterende des Pons abgebildet. In der ZIEHEN-Serie lag die Kreuzung ebenfalls an dieser Stelle, obgleich hier die kreuzenden Fasern mehr verflochten waren. In der SYMINGTON-Serie liesz sich die Kreuzung leider nicht mit Sicherheit ermitteln.

Zum Schlusse wollen wir die wichtige Frage erörtern, ob ein Teil der Fasern der lateralen Pyramiden, wie es die Autoren einstimmig behaupten, schon vor dem Rückenmark seine Endstätte erreicht.

Nach FUSE (1926a) endet sogar ein sehr beträchtlicher Teil der betreffenden Fasern schon in der Oblongata und zwar an verschiedenen Stellen: im Kerne der absteigenden V-Wurzel durch seine ganze Länge, in der oberen Olive, im motorischen VII-Kern und im Seitenstrangkern. Auch ABBIE ist der Meinung, dasz sein Tractus temporo-trigeminalis schon vor dem Rückenmark teilweise sein Ende findet.

Eine Angabe ZIEHENS (1908, p. 800) scheint hierzu in Widerspruch zu stehen. Beim erwachsenen Tiere verwischt sich nach hinten zu der Unterschied zwischen Pyramide und V-Wurzel mehr und mehr. Für das jugendliche Tier, das ZIEHEN untersuchte und das oben schon erwähnt wurde, gibt er nun an, dasz er infolge des Unterschiedes in der Markreifung feststellen konnte, dasz die V-Wurzel sich bald erschöpft, während die Pyramide ohne Fasereinbusze in das Rückenmark übergeht. Doch sagt derselbe Autor etwas später (1908, p. 807), dasz die Zonalbündel mit dem Nucleus laciniatus in ausgiebiger Verbindung stehen.

Unsere Meinung in dieser Sache ist, dasz es in Markfaserpräparaten unmöglich ist mit Sicherheit die Endstätte der Pyramidenfasern festzustellen. Dazu wären Silberpräparate und zumal Experimente erforderlich.

Dasz ein guter Teil der Pyramidenfasern aber schon in der Oblongata endet, geht wohl mit Sicherheit hervor aus der beträchtlichen Verminderung an Umfang, die die Pyramiden auf ihrem Wege zum Rückenmarke erleiden (vgl. Fig. 5 mit Fig. 6 und 7). Nach unserem Dafürhalten findet diese Endigung hauptsächlich statt im Kern der absteigenden V-Wurzel durch seine ganze Länge. Dieses Verhalten mag sogar verantwortlich sein für die orale Lage der Kreuzung der lateralen Pyramiden und ihre Vereinigung mit der absteigenden V-Wurzel.

Obgleich die Monotremen die primitivsten jetzt lebenden Säuger sind, glauben wir nicht, dasz die Lage der lateralen Pyramiden und ihre vordere Kreuzung ursprüngliche Merkmale darstellen. Wir wollen eher annehmen, dasz diese Merkmale, wie so manche andere Eigentümlichkeiten der Monotremen, sekundär erworben sind im Zusammenhang mit der spezialisierten Lebensweise dieser Tiere. Die starke Entwicklung der

taktilen Sensibilität des Kopfes scheint uns hier verantwortlich. Leider sind die Pyramidenbahnen von *Ornithorhynchus*, wo der Trigeminus noch viel stärker entwickelt ist als bei *Echidna*, zu ungenügend bekannt, um diese Ansicht auf ihre Richtigkeit zu prüfen.

Zusammenfassung.

Nach unserer Auffassung hat *Echidna* zwei Pyramidenbahnen, eine mediale ungekreuzte und eine laterale gekreuzte. Die mediale Bahn, vielleicht schon von ZIEHEN (1897a, '99, 1908) und KÖLLIKER (1901) gesehen, doch deren Anwesenheit später von FUSE (1926a) verneint wurde, ist am ersten von ABBIE (1934) mit Sicherheit beschrieben worden. Sie liegt an der gewöhnlichen Stelle, ventral vom Lemniscus medialis, ist winzig und aus feinen, schwach markhaltigen Fasern zusammengesetzt. Wir verloren das Bündel in der Höhe des oralen Anfangs des Hypoglossuskernes aus dem Auge. Die Kreuzung, die ZIEHEN, KÖLLIKER und ABBIE am Anfang des Rückenmarkes beschreiben, halten wir mit FUSE für ein Fasersystem der Commissura ventralis. Oral geht das Bündel wahrscheinlich aus dem gleichseitigen Pes pedunculi hervor. Wir konnten es aber nicht wie ABBIE als einen besonderen Anteil des Pes nach vorne verfolgen.

Die lateralen Pyramiden (Fig. 4—7) gehen hervor aus dem Pes pedunculi, kreuzen sich im Hinterende des Pons und schlieszen sich dann der Auszenseite der absteigenden Trigeminuswurzel an, womit sie zusammen zum Rückenmark ziehen. Hier enden sie im dorsalsten Teil des Seitenstranges. Sie wurden 1901 von KÖLLIKER entdeckt, aber nicht als Pyramiden erkannt und als Zonalbündel bezeichnet. Auch ZIEHEN (1908) und ABBIE (1934), der sie zum Tractus temporo-trigeminalis umtaufte, betrachten sie als eine Bahn, die nur *Echidna* zukomme. FUSE (1926a) deutete sie zum erstenmale als Pyramiden. Nach ABBIE entstehen seine Tractus temporo-trigeminales (unsere lateralen Pyramiden) zusammen mit den temporo-pontinen Fasern ventral und kaudal vom Sulcus α ELLIOT SMITHS aus Rinde vom Typus IV SCHUSTERS. Nach unseren Beobachtungen gehen die Pyramiden aber dorsal und oral von dieser Furche aus Rinde vom Typus III hervor. Ein beträchtlicher Teil der Fasern der lateralen Pyramiden scheint im Kerne der absteigenden V-Wurzel zu enden. Dies mag eine Erklärung abgeben für die orale Lage der Kreuzung dieser Bahnen sowie für ihre Vereinigung mit der V-Wurzel.

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Medicine. — *Chronic experimental catatonia, produced by intermediate products of metabolism.* — *Indolethylamine.* By F. J. NIEUWENHUYZEN. From the neurophysiological laboratory (Dr. H. DE JONG) of the neurological clinic of the University of Amsterdam (Prof. B. BROUWER). (Communicated by Prof. B. BROUWER).

(Communicated at the meeting of October 31, 1936).

Historical Review.

Until 1935 we succeeded to produce only acute experimental catatonia in test animals. We refer herefore to the extensive series of experiments of H. DE JONG, H. BARUK and other collaborators. We mention e.g. catatonia produced by different chemicals, hormones, auto-intoxication, surgical lesions of the brain, asphyxia and faradic stimulation of the cranium. Most of these agentia produced only *acute* catatonic phenomena. The catatonia caused by auto-intoxication as a consequence of ligation of the arteria hepatica or by artificial ileus, gave a *chronic* catatonic state¹). Previously we have not examined the toxin or toxins, which play a causative role.

¹) H. DE JONG, D. J. KOK, A. GEESINK and F. J. NIEUWENHUYZEN: "Experimental catatonia, produced by auto-intoxication I. Experimental catatonia after artificial obstruction of the lumen of the intestine". — Proc. Royal Acad., Amsterdam, 38, 332—335 (1935).

A. GEESINK, H. DE JONG and F. J. NIEUWENHUYZEN: "Experimental catatonia produced by auto-intoxication II. Experimental catatonia after ligation of the arteria hepatica propria". Proc. Royal Acad., Amsterdam, 39, 701—707 (1936).

Now after years of research, it appears to us that the biological product, indolethylamine, is able to produce a chronic state.

EWINS and LAIDLAW (1910)²⁾ obtained indolethylamine in vitro by the action of putrefactive bacteria on tryptophane. They considered it probable that indolethylamine was produced in the intestinal tract under physiological and certain pathological conditions. SULLIVAN (1922)³⁾ succeeded in isolating indolethylamine from the urine of patients suffering from pellagra. It is remarkable that in pellagra just as in schizophrenia, stuporous and catatonic states occur (MÜLLER 1933)⁴⁾.

If indolethylamine is perfused through the liver in vitro, it is converted into indoleacetic acid. When ingested indoleacetic acid is excreted in combination with glycine as indoleaceturic acid. (EWINS and LAIDLAW 1913)⁵⁾. Schizophrenic patients often suffer from disfunction of the liver (KLIPPEL 1892, SCHRIJVER and SCHRIJVER—HERZBERGER 1924, LINGJAERDE 1934, JØRGEN RAVN 1935)⁶⁾; so the question arises if in these cases metabolism of indolethylamine may also be disturbed.

Experiments.

After intravenous injection of 100—150 mG of indolethylaminehydrochloride in 3 KG cats, autonomic phenomena occurred (salivation and narrowing of the pupils), combined with negativism and catalepsy. The animals could be pushed forwards "en bloc", and after the shoving had ceased they took again their normal position. When hung to lattice-work, the animals remained hanging in this position. These phenomena persisted several days. The experiments were made on 7 cats.

Protocol. Adult normal cat E.V. (3,3 KG). The animal is clinically healthy.

February 9th, 1935.

11.40 A.M. Intravenous injection of 100 mG indolethylaminehydrochloride.

²⁾ EWINS, A. J. and LAIDLAW, P. P.: "The syntheses of 3- β -aminoethylindole and its formation from tryptophan". Proc. Chem. Soc. London, 26, 343 (1910).

³⁾ SULLIVAN, M. X.: "Indolethylamine in the urine of Pellagrins". Journ. Biol. Chem. 50, 39 (1922).

⁴⁾ MÜLLER, H.: „Zur Aetiologie der Pellagra". Zeitschr. f. d. ges. Neur. u. Psych. 146, 429—438 (1933).

⁵⁾ EWINS, A. J. and LAIDLAW, P. P.: "The fate of indolethylamine in the organism". Bioch. Journ. 7, 18—25 (1913).

⁶⁾ KLIPPEL: Arch. gén. de méd. 2, 173 (1892).

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F. J. NIEUWENHUYZEN: CHRONIC EXPERIMENTAL CATATONIA,
PRODUCED BY INTERMEDIATE PRODUCTS OF METABOLISM. —
INDOLETHYLAMINE.



Fig. 1.



Fig. 2.

Fig. 1 and 2: Indolethylaminecatalepsy. The animal remains actively in passively given postures.

- 12.00 A.M. 50 mG of indolethylaminehydrochloride injected subcutaneously.
- 12.02 P.M. *Salivation and narrowed pupils.* The cat shows *diminished motoric activity* and gives a catatonic impression. The animal is also *negativistic*: it can be pushed forwards like a block and resists change of attitude.
- 12.20 P.M. The animal walks slowly in its cage.
- 12.37 P.M. When placed on the edge of a table, the animal remains sitting with closed eyes in flexed position more than an hour, but reacts on stimulations by looking upwards.
- 1.57 P.M. The cat still sitting in a flexed posture, is placed on the rung of a chair. It shows a strong *catalepsy* and remains hanging on the chair (fig. 1 and 2).

February 11th, 1935.

Placed on the table, the animal assumes the same *flexed posture* as on February 9th. The animal may be pushed forwards like a block (*Negativism*). If hung to lattice-work, the cat slowly takes a normal position. When the forepaws have come to the ground they remain *crossed*, the hindlegs in abduction (*Catalepsy!*). When the cat walks in its cage, the forepaws remain sometimes in crossed posture for a long time.

February 13th, 1935.

The cat is still cataleptic. When hung to lattice-work, the animal remains hanging in this posture.

February 15th, 1935.

Increased motoric activity.

Summary.

Intravenous injection in cats of indolethylaminehydrochloride caused the complete syndrome of experimental catatonia: catalepsy, negativism and autonomic phenomena.
